AN

ELEMENTARY TREATISE

ON

HYDROSTATICS,

FOR THE USE OF JUNIOR UNIVERSITY STUDENTS.

BY

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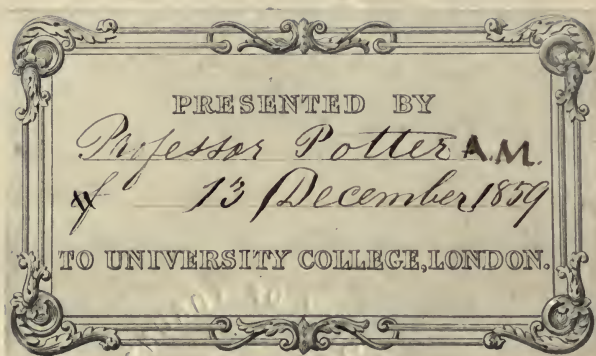PROFESSOR OF NATURAL PHILOSOPHY AND ASTRONOMY
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PREFACE.

THE present Treatise on Elementary Hydrostatics is written to supply a text-book for the Author's Junior Mathematical Class of Natural Philosophy, and to include the various propositions which can be solved without the Differential Calculus.

Within the prescribed limits it has been his wish to make it as comprehensive as possible without entering into the bounds of practical hydraulics, more than is considered appropriate to a theoretical treatise.

The Author has endeavoured to meet the wants of Students who may look to hydraulic engineering as their profession, as well as those who learn the subject in a course of scientific education.

He hopes that in preparing a text-book for his own teaching, he has also produced a treatise which may be useful to others similarly engaged in tuition.

LONDON, 1859.

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CHAPTER VII.

HYDROSTATICS.

INTRODUCTION.

In the Mechanical Sciences, which are called *Hydrostatics* and *Hydrodynamics*, the matter which is subject to the action of forces is said to be in a state of *fluidity*.

In solid bodies the particles are held together by the forces of *cohesion;* but in fluids, the effect of an increase of *caloric* (the cause of heat) has been to take away this force of cohesion, so that the atoms of a perfect fluid are put in motion amongst each other on the application of the slightest force. We must consider that when at rest, the atoms of such fluids are in a state of neutral equilibrium under the actions of the internal and external forces to which they are subject; so that any additional force applied to an atom puts it in motion. We must remember that as bodies expand on being heated and contract in volume on being cooled, the ponderable hard parts of the atoms must be separated by considerable intervals, and possess in any given state atmospheres of caloric and other imponderable fluids, which produce the repulsive forces counteracting the attractive forces of the hard nuclei for each other. So that the actual state of a quiescent body must be that of equilibrium amongst its atoms under the action of opposing molecular or atomic forces.

In strictness the *elastic* or aeriform fluids alone fulfil the condition of perfect fluidity, for the liquids or dense fluids possess the *attraction of aggregation* by which small isolated portions of them collect in spherical drops, and some degree of force is

required to separate the parts of such drops which thus exhibit
a force of adhesion. In some problems this attraction of aggre-
gation or adhesion is an essential property to be considered; but
in the greater number of cases the properties of fluidity common
to liquids and gases alone affect the results. This is the case
even for the viscid fluids, such as tar, syrups, &c., which require
time to arrive at their state of equilibrium. The characteristic
state of fluid bodies, in contradistinction to that of solid bodies,
is that their particles are capable of motion amongst each other
on the application of the slightest force.

The characteristic distinction of solids and fluids as to the
transmission of pressure evidently arises from the mutual rela-
tions of their constituent atoms, so that solid bodies only trans-
mit pressure in the direction of its action; but fluids transmit
pressure equally in all directions from the absence of stable rela-
tions between contiguous atoms.

Fluids are subdivided into *non-elastic* fluids or liquids; and
elastic fluids or gases and vapours.

Definitions. The mass of a portion of fluid is the *quantity of
matter* which it contains as measured by its *inertia*, and is pro-
portional to its weight at the same place. The weight of a
body, as in dynamics, is the pressure produced by it under the
action of the force of gravity.

Def. The density of a body, solid or fluid, is the relation of
the quantity of matter it contains to its bulk, and is measured
by the mass or *quantity of matter in a unit of volume*, when uni-
form. It is however frequently expressed by reference to the
density of some fluid taken as a standard. Thus we say the
density of mercury is about $13\frac{1}{2}$ times that of water; the density
of silver is about $10\frac{1}{2}$ times that of water. In these cases water
is evidently taken as the standard fluid. Sometimes the densi-
ties of the gases are referred to a standard gas, as when we say
that the density of hydrogen gas is only $\frac{1}{14}$th that of atmospheric
air, and then we take atmospheric air for our standard fluid.
The two methods of expressing the density of a body are easily
convertible the one to the other in the mathematical results.

Def. The specific gravity of a body is *the weight* of a unit of volume, and is generally expressed, like density, by reference to a standard fluid. Thus, when we say that the weight of a cubic foot of iron is nearly eight times that of a cubic foot of water, we refer the specific gravity of iron to that of water as a standard. Also we know the actual weight of a given volume of a body when we know its specific gravity, since a cubic foot of distilled water weighs 1000 ounces avoirdupois at the temperature 60° Fahrenheit very nearly.

The tables of densities and specific gravities are evidently identical when the same standards are taken. A table will be found at the end of this volume.

The term unit of pressure in hydrostatics has a different meaning to that which it has in statics. In statics, pressure being generally represented by weight, the unit of pressure is the unit of weight which is taken, as one ounce, one pound, or one ton; but in hydrostatics we have to consider the pressures of fluids upon surfaces, and then we call the pressure upon a *unit of area* of the surface, *the unit of pressure*, when it is a uniform pressure. For example, if a cistern containing water has its base *horizontal,* then the pressure on every square foot of the base is the same, and the pressure on any area of it is proportional to that area; so that if the pressure on a square foot is known, the pressure on any given part of the base is known. In this example a square foot is taken as the unit of area; in other cases we might take a square inch, or square yard, as the unit of area; and we can pass easily in calculation from one unit to another, as the problem may require.

In this manner, if p be put for the unit of pressure, A the area on which the pressure is P, and the pressure is uniform or constant, we have

$$P = p \cdot A.$$

When the pressure is not constant, then p is not constant, and this formula does not apply; and the value of P requires to be found from the rules of the science as investigated further on. This will be the case whenever the pressure is required upon a given area of the *vertical side* of a cistern containing water for

example; where the pressure increases with the depth below the surface of the water.

The force of gravity is expressed in hydrostatics in the same manner as in dynamics, being measured by the velocity produced by its action in one second of time; and we put force of gravity $= g = 32\cdot19$ feet velocity per one second. In hydrodynamics this number has to be employed often, as it has in dynamics; but in hydrostatics the expressions which contain it are easily changed to others in terms of weights, since in dynamics weight equals gravity multiplied by the mass or $w = g \cdot m$.

The mass is proportional to the volume of a body when the density is constant, or if we put m for the mass when the volume is V and the density is ρ, we have

$$m \propto V \text{ when } \rho \text{ is constant};$$

and again, the mass varies as the density when the volume is constant, or

$$m \propto \rho \text{ when } V \text{ is constant};$$

so that by the rules of Algebra, when V and ρ may both vary, we have

$$m \propto \rho \cdot V;$$

and if C be some constant, $m = C \cdot \rho \cdot V$.

To find the value of C, let m_1, ρ_1, V_1 be some given simultaneous values of m, ρ, and V, then

$$C = \frac{m_1}{\rho_1 \cdot V_1}.$$

When we put $C = 1$, as is usual, and then have

$$m = \rho \cdot V,$$

we see that the units of mass, density, and volume, are not independent of each other, but the mathematical expressions must be brought to a recognized form when we want to perform actual computations; and this is easily accomplished, for we have

$$w = g \cdot m$$
$$= g \cdot \rho \cdot V;$$

and therefore, in any formula where $g . \rho . V$ occurs, we may replace it by the weight w, of which the unit of measure will be known from the data of the question: also, conversely, in order to solve any question, we may replace w by its equivalent $g\rho V$, when the solution turns upon the density and the volume. Examples of these will be found further on.

It was formerly thought that liquids were absolutely non-elastic; but Canton proved, in the year 1762, that they diminished slightly in bulk under pressure, and recovered their original volume when the pressure ceased; and this has been confirmed by Perkins, Colladon and Sturm, Œrsted, Regnault, and others. The law of Canton is this, that *the diminution in volume of a liquid is proportional to the pressure to which it is subject;* also, that the *amount of diminution* under a given pressure is different for *different liquids.* The following table contains his results when the barometer stood at $29\frac{1}{2}$ inches, and the thermometer stood at 50° Fahrenheit.

	Compression of a volume unity under the pressure of the atmosphere.	Specific gravity.
Spirit of wine	·000066	·846
Oil of olives ...	·000048	·918
Rain-water	·000046	1·000
Sea-water	·000040	1·028
Mercury	·000003	13·595

We see that the compressibility of liquids is so small that it is only in particular cases it needs to be taken into account. The law of Canton is expressed in a formula as follows. Let V be the original volume of the liquid and the density ρ, V' the volume under a pressure p measured in atmospheric pressures upon a unit of area, and the density ρ'; let c be the compressibility or the value of the numbers in the second column of the table, that is, when $p = 1$.

Then $\qquad V - V' = V.c.p,$

$\qquad\qquad$ or $\qquad V' = V(1 - cp),$

and since $\qquad V'.\rho' = V.\rho,$ we have $\rho' = \dfrac{\rho}{1 - cp}.$

The volumes of aeriform fluids change greatly with changes of pressure and temperature, and their peculiar properties involve in a very great degree their relations to heat. The laws to which they are subject will be found in the chapter treating of gases and vapours. By a vapour is meant an elastic fluid which is *easily reduced* to the liquid state by the application of cold or pressure, or both. We say *easily* reduced to the liquid state, for many of the gases have been reduced to the liquid and some to the solid state.

Amongst those which have been brought to the solid state are carbonic acid, ammonia, cyanogen, euchlorine, sulphureted hydrogen, sulphurous acid, hydriodic acid, nitrous oxide, &c. Others, as muriatic acid and olefiant gas, have been reduced to the liquid state; whilst the following were found by Dr Faraday to remain in the gaseous state at the temperature 166° below the zero of Fahrenheit's thermometer.

Hydrogen $\qquad$ at 27 atmospheres pressure
Oxygen $\qquad$ ... 27 ....
Nitrogen $\qquad$ ... 50 ....
Nitric oxide $\quad$ ... 50 ....
Carbonic oxide ... 40 ....
Coal-gas $\qquad$ ... 32 ....

CHAPTER I.

In the introductory chapter it was stated that fluids at rest transmit pressure equally in all directions, on account of the state of neutral equilibrium which exists for each constituent atom of the fluid, arising from their mutual pressures upon each other, and being the results of the internal and external forces to which the body is subject. The experimental proof of this property is as follows.

PROP. 1. *To describe the experimental proof that fluids transmit pressures in all directions.*

Let $ABCDE$ represent the horizontal section of a close vessel filled with fluid, having pipes in the sides with tight-fitting pistons, and the fluid filling the pipes up to the pistons, of which the centers are in the horizontal section of the figure 1.

Fig. 1.

Now if any pressure is applied to any one of the pistons, then pressures must be applied also to each of the others to keep them in their places; and the pressures must be proportional to the areas of the pistons respectively, so that the unit of pressure is the same for all, and likewise for all parts of the surface of the vessel; or fluids transmit pressure equally in all directions.
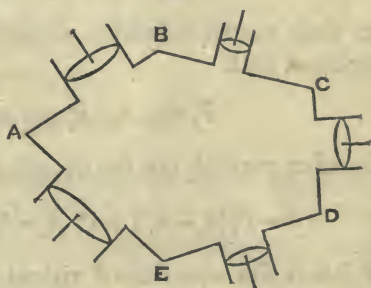
Prop. 2. *To shew that the equation of virtual velocities holds good for fluids in equilibrium.*

In the figure 1, let a_1, a_2, a_3, &c. be the areas of the pistons, and h_1, h_2, h_3, &c. their distances respectively in the pipes from the body of the vessel.

Let V be the volume of fluid in the vessel itself, so that the whole volume in the vessel and pipes

$$= V + a_1 . h_1 + a_2 . h_2 + a_3 . h_3 + \&c.$$

After the pistons have received simultaneous displacements, let h_1', h_2', h_3', &c. be their distances respectively in the pipes.

And since the volume of fluid is unchanged it is

$$= V + a_1 . h_1' + a_2 . h_2' + a_3 . h_3' + \&c.,$$

or subtracting this value from the former one, we have

$$0 = a_1 (h_1 - h_1') + a_2 (h_2 - h_2') + a_3 (h_3 - h_3') + \&c.\dots\dots(1).$$

Now if P_1, P_2, P_3, &c. are the pressures applied to the pistons respectively in equilibrium, then

$$(h_1 - h_1'), \ (h_2 - h_2'), \ (h_3 - h_3'), \&c.$$

are their virtual velocities respectively; and if p is the unit of pressure, then

$$P_1 = p . a_1, \ P_2 = p . a_2, \ P_3 = p . a_3, \&c.;$$

therefore multiplying the equation (1) by p, we have

$$0 = P_1 (h_1 - h_1') + P_2 (h_2 - h_2') + P_3 (h_3 - h_3') + \&c.,$$

which is the equation of virtual velocities; and if some of the pistons are moved inwards, others must be moved outwards, or some of the terms will be positive and others negative.

Prop. 3. *To find the pressure upon a horizontal plane surface within a fluid of uniform density, and subject to the action of gravity.*

Let AB be the horizontal plane surface and its area a, upon which the pressure P is to be found.

Fig. 2.

Suppose a vertical prism of the fluid $ABCD$ above AB up to the surface CD of the fluid, and whose base is AB, to be separated from the rest of the fluid by an imaginary rigid film, which would not alter the state of the fluid; then if the outside fluid were removed, we see that the whole weight of the internal fluid must be supported by the base AB, since no part would be supported by the vertical sides of the film; and the pressure upon AB would be the weight of the fluid equal in volume to the prism.
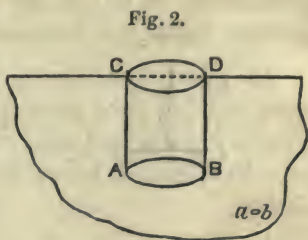
Since before the external fluid was supposed to be removed there was equilibrium; therefore the pressure on the under side of the area AB equals the pressure on the upper side of it, and each equals the weight of fluid of the volume of the prism.

Let the depth $A.C = z$, ρ the density of the fluid, and p the unit of pressure on AB; then the volume of the prism $V = a.z$,

$$\text{and } P = g\rho V = g\rho az.$$

Also since the pressure is uniform, $\therefore P = p.a$,

$$\text{and } p = g\rho z,$$

or the pressure varies directly as the depth z.

If the area had been indefinitely small, as ab in the figure, and equal to α, we should have had the pressure upon it in like manner equal to $p.\alpha$. But when the area is indefinitely small it will be all sensibly at the same depth when not horizontal, so that on the surface of a body in a fluid, or on the surface of the containing vessel, the pressure on any indefinitely small area is $p.\alpha = g\rho z.\alpha$.

COR. 1. It is evident the pressures are equal at all equal depths, and since fluids transmit pressure equally in all direc-

tions, therefore in equilibrium the upper free surface where the pressure vanishes is horizontal.

In viscid fluids the surface will become horizontal only in some sensible time after it has been disturbed; also in fluids generally waves will arise when the fluid is disturbed, and they must have ceased before the fluid can be said to be in equilibrium.

Cor. 2.   Within a fluid which is in equilibrium we may suppose imaginary films to separate certain portions from the rest without disturbing the respective parts, and all the parts of the surface would remain in the same horizontal plane. Hence when a fluid is contained in a system of vessels which communicate with each other, the surfaces of the fluid in all the vessels are in the same horizontal plane.

Fig. 3.

If fig. 3 represents a set of vessels of different forms communicating by a pipe AB, then the surfaces C, D, E, F, in the separate vessels will be all in the same horizontal plane.   This in common language is stated by saying that '*fluids always find their level*,' or tend to do so.
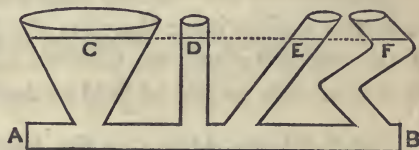
This is shewn also by taking a horizontal plane in the pipe AB, at which the pressures must be equal in equilibrium, and the heights of the surfaces above it the same, whatever be the form of the vessels.

Cor. 3.   From this we see that a *level surface*, when of small extent, is a *horizontal plane;* but when large, as the surface of a lake, it has the curvature of the earth's surface. These must be examined when the uses of the spirit-level are discussed.

Cor. 4.   Since the pressure upon a horizontal area depends upon the magnitude of the area, and upon the depth below the surface of the fluid, the pressure on the bases of vessels resting upon a horizontal table is the same when the bases are of the same area, and the depth of the fluid is the same *whatever be the quantity in the vessel.*

Let AB be the base of a
cone ABC, or of a cylinder
$ABDE$, or of an inverted frus-
tum of a cone $ABGF$, or any
other form of vessel; then the
pressure on AB is the same
whichever of the vessels be
used, if the fluid be at the same height in all.

Fig. 4.
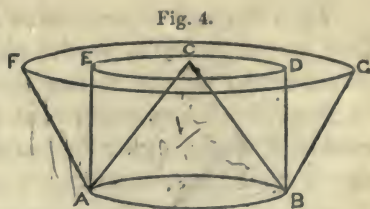
The pressure on the base AB of the cylinder $ABDE$ is the
weight of the contained fluid, since the vertical sides can sup-
port no part of the weight; but the cone being one-third the
volume of the cylinder, the pressure on its base, when filled
with fluid, is three times the weight of the fluid it contains.
In the inverted frustum of a cone the pressure is evidently less
than the weight of the contained fluid.

The pressure upon the table in each case is the weight of the
vessel and the contained fluid. The pressure on the inner sur-
face of the base of the cone filled with fluid, neglecting the
weight of the vessel itself, is three times that of the outer sur-
face of it upon the table. The resultant vertical pressure will
be found discussed in Prop. 5, in explanation of this circum-
stance.

PROP. 4. *To shew that the horizontal pressures upon the sur-
face of a vessel containing fluid are in equilibrium with each
other.*

Let PQ be any indefinitely small *horizontal* prism in figures
5 and 6, taken in vessels containing fluid.

Fig. 5.
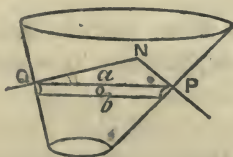
Fig. 6.
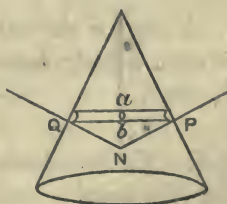
Then the unit of pressure at P and Q will be the same, since they are at the same depth; also the pressure is in every case perpendicular to the surface at each point where there is equilibrium; for if any component pressure parallel to the surface existed, it would set the fluid in motion, contrary to the supposition of equilibrium existing.

Then if PN, QN are the normals to the surfaces at P and Q respectively, the angles they make with the axis of the prism are the same as the angles between the perpendicular section ab of the prism and the oblique sections in which it meets the surfaces of the vessels at P and Q, since these sections are perpendicular respectively to the former directions.

Let PN make an angle θ with the axis of the prism PQ.

Let $\alpha =$ the perpendicular section ab of the prism PQ in the figures.

$\alpha' =$ the oblique section in which it meets the surface of the vessel at P.

$\alpha'' =$ that at Q.

We have $\alpha = \alpha' \cos NPQ = \alpha'' \cos NQP$ by the property of projections.

Then p being the unit of pressure at P and Q, $p\alpha'$ is the pressure on the oblique section at P, and its horizontal component is $p\alpha' \cos \theta = p\alpha$, since $\dfrac{\alpha}{\alpha'} = \cos \theta$.

Similarly the horizontal component of $p\alpha''$ at Q is $p\alpha$, and the same result will arise in whatever horizontal direction the prism be taken, therefore the horizontal pressures are always in equilibrium with each other.

COR. A similar method of proof shows that the horizontal pressures upon a *body immersed in a fluid* are in equilibrium with each other.

PROP. 5. *To investigate the resultant vertical pressures on the surfaces of a vessel containing fluid.*

Let PP' be an indefinitely small vertical prism meeting the horizontal base AB of the vessel ABC, which is filled with fluid in the indefinitely small area α at P'; then if α' is the oblique area in which the prism cuts the surface of the vessel at P, and the angle between the normal PN, and the horizontal line PQ is θ, as in the last Prop., we have now

Fig. 7.
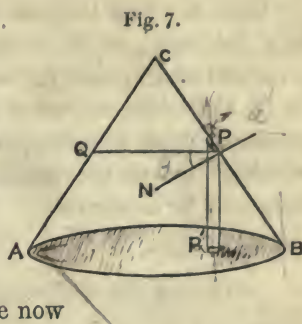
$$\frac{\alpha}{\alpha'} = \cos NPP' = \sin \theta.$$

Now the pressure on the oblique area at $P = p\alpha'$, and its upward vertical component

$$= p\alpha' \cos NPP'$$
$$= p\alpha' \sin \theta$$
$$= p\alpha$$
$$= g\rho z . \alpha.$$

z being the depth of PQ below the surface, let z' be the depth of P' below the surface of the fluid;

Then the pressure on the area α at P' is downwards, and

$$= g\rho z' . \alpha.$$

The difference of the downward pressure at P', and the upward pressure at P,

$$= g\rho\alpha (z' - z)$$

$=$ weight of the fluid of the volume of the prism PP'.

The same result holds for all indefinitely small *vertical* prisms which can be taken in the fluid; and the resultant of all the vertical pressures, upwards and downwards, equals the weight of the fluid contained in the vessel, of whatever form it may be.

Cor. 1. If a body be immersed in a fluid, such as PQP' (fig. 8), and a vertical small prism PP' were taken in it, the difference of the vertical pressures upon the oblique areas in which it meets the surface of the body will be as above

Fig. 8.

$$= g\rho\alpha\,(z' - z)$$

= the weight of the fluid of the volume of the prism PP'.

Now the same being true for all other small vertical prisms which can be taken in the body, the resultant of the vertical pressures acting upon the body equals the weight of an equal volume of the fluid to it, or which it displaces, and acts vertically upwards.

The resultant acts vertically downwards through the center of gravity of the contained fluid in the first case (fig. 7), and upwards through the center of gravity of the fluid displaced in the latter case (fig. 8), since the vertical pressures are systems of parallel forces represented by the weights of the prisms of fluid.

Cor. 2. If the body were only partly immersed, we should have the prisms terminating in the plane of the surface of the fluid, and $z = 0$ for some parts of the body; but the sum of the prisms of fluid would make up the whole fluid displaced, and the resultant pressure would be the weight of the fluid displaced, as before.

Cor. 3. That the *resultant pressure* of a fluid upon a body, *wholly* or *partly* immersed in it, equals the weight of the fluid displaced by it, and acts vertically upwards through the center of gravity of the fluid displaced, is easily shown at once by supposing in fig. 8 and fig. 9, that before the body was immersed in the fluid, the part displaced was separated from the rest by an imaginary rigid film; then if the exterior fluid were removed, that within the film would gravitate vertically downwards with a pressure equal to its weight acting through its center of gravity; and before the exterior fluid was removed, there being equili-

brium, this downward pressure was balanced by an equal and opposite upward pressure from the exterior fluid. The exterior fluid would produce evidently the same pressure upon *the surface* of the body, wholly or partly immersed in it, that it does upon the imaginary film of the same form and position; and hence we conclude, that *the resultant pressure of a fluid upon a body, wholly or partly immersed in it, equals the weight of the fluid displaced, and acts vertically upwards through the center of gravity of the fluid displaced.*
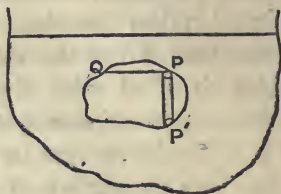
PROP. 6.   *To find the accelerating force of relative gravity with which a body immersed in a fluid ascends or descends.*

Referring to fig. 8, the resultant moving force acting upon the body is evidently the difference of its weight acting downwards through its center of gravity, and of the vertical pressure of the fluid upwards through the center of gravity of the fluid displaced. Now if the body is homogeneous, or its density uniform, these will be opposite vertical forces acting through the same point, the center of gravity of the body or fluid displaced in all positions of the body; but if its density is variable, there will be only certain positions of the body when its center of gravity and that of the fluid displaced are in the same vertical line, and in the other positions there will be a resultant pressure which generally will produce motion of rotation as well as translation; and thus a body ascending or descending in a fluid may at the same time have an oscillatory motion.

Taking now the case of the homogeneous body only,

let ρ be the density of the fluid,

ρ' .... body,

V' be the volume of the body;

then the weight of the body $= g\rho'V'$, and the weight of the fluid displaced $= g\rho V'$.

Now supposing the body to descend in the fluid, we have the resultant moving force $=$ weight of the body $-$ weight of

the fluid displaced $= g V'(\rho' - \rho)$,

and the mass of the body $= \rho'V'$;

Δ

∴ the accelerating force of relative gravity

$$g' = \frac{\text{resultant pressure}}{\text{mass moved}}$$

$$= \frac{g\,V'\,(\rho' - \rho)}{\rho'\,V'}$$

$$= g\left(1 - \frac{\rho}{\rho'}\right),$$

which is a fractional part of gravity g, depending on the values of ρ and ρ'.

We have supposed ρ' greater than ρ, or the body to descend; and if ρ' were less than ρ, we should have

$$g' = -g\left(\frac{\rho}{\rho'} - 1\right),$$

and the body would ascend in the fluid.

If $\rho' = \rho$, then $g' = 0$, and the body would remain at rest any where in the fluid.

The case of the air-balloon ascending and descending in the atmosphere fulfils these three conditions. When the balloon, its contained gas, and appendages, weigh less than the air they displace, the balloon rises; when they weigh more than the air displaced, it descends.

PROP. 7. *To find the conditions that a body may float in equilibrium at the surface of a fluid.*

In order that two forces may balance, they must act through the same point, be equal in magnitude and opposite in direction; therefore, in the case of the equilibrium of floating bodies, the weight of the body being supported by the upward fluid pres-

Fig. 9.

sure, we have, by Cor. 2, Prop. 5, the two conditions to be fulfilled as follows.
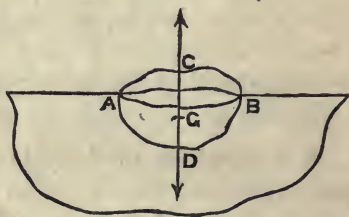
(1) The weight of the body equals the weight of the fluid it displaces.

(2) The centers of gravity of the body and the fluid displaced must be in the same vertical line.

DEF. The plane in which the surface of the fluid cuts the floating body is called the *plane of floatation*.

PROP. 8. *To find how deep a given body will sink in a given fluid of greater density than itself, when floating in equilibrium.*

In fig. 9, let V be the volume of the fluid displaced, ρ its density, V' the volume of the body, and ρ' its density, then the first of the conditions of the last proposition gives us

$$g\rho V = g\rho' V',$$

and therefore $\dfrac{V}{V'} = \dfrac{\rho'}{\rho}$.

The volume V must be also taken so as to fulfil the second condition, or the plane of floatation AB must be perpendicular to the line joining the centers of gravity of the body and the fluid displaced.

In a few simple cases the computation for the value $V = V' \dfrac{\rho'}{\rho}$ is easy; but in the more interesting cases of floating bodies it is more complicated.

Examples will be found at the end of the chapter.

PROP. 9. *When the density of the fluid increases as the depth, to find the weight of a body at a given depth.*

Let ρ be the density at the surface of the fluid, σ the increase of density at a depth unity, and therefore the density at a depth z will be $\rho + \sigma z$.

Let ρ' be the density of the body, and V' its volume, then its weight at the depth $z =$ weight of the body $-$ the weight of the fluid it displaces

$$= g\rho' V' - g(\rho + \sigma z) \cdot V'$$
$$= gV'\{\rho' - (\rho + \sigma z)\},$$

and the body will float in the fluid at a depth z, such that

$$z = \frac{\rho' - \rho}{\sigma}.$$

PROP. 10. *A floating body being slightly displaced vertically from its position of equilibrium, to find the accelerating force causing it to return to the position of equilibrium.*

Let AB be the original plane of floatation, of which the area is A; let ab be the level of the surface of the fluid, and x the depth of AB below ab.

Fig. 10.
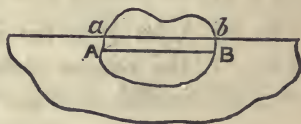
Then the volume of fluid displaced more than in equilibrium will be Ax, nearly, and the weight of this volume is the moving force bringing the body back to its position of equilibrium. Let as before $\rho =$ density of the fluid, $V =$ volume of fluid displaced in equilibrium, $\rho' =$ density of the body, and $V' =$ its volume.

Then the accelerating force bringing the body to its original position

$$= \frac{\text{weight of the volume } Ax \text{ of the fluid}}{\text{mass of the body}}$$

$$= \frac{g\rho Ax}{V'\rho'}, \qquad \text{and } V'\rho' = V\rho,$$

$$= \frac{g\rho Ax}{V\rho}$$

$$= g\frac{A}{V}x,$$

which varies with the depth x and area A, and changes sign when AB is above ab.

Calculations for the motion of the body from this formula are only approximate, because the disturbance of the floating body will give rise to waves on the surface of the fluid, which are not taken into account.

Cor. If the body were floating in a
vessel of finite extent, and the area of the
surface of the fluid in the vessel were A',
the rise of the surface of the fluid in the
vessel, by the depression of the body,
would be sensible. Let x' be the rise of
the surface of the fluid to ab, fig. 11, and $x + x'$ the height of ab
above AB,

Fig. 11.
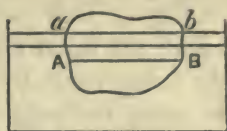
$$\text{then } (A' - A)x' = Ax, \text{ and } x' = \frac{A}{A' - A} \cdot x,$$

and we have the accelerating force

$$= \frac{\text{weight of the volume } A(x + x') \text{ of the fluid}}{\text{mass of the body}}$$

$$= \frac{g\rho A x \left(1 + \dfrac{A}{A' - A}\right)}{V'\rho'}$$

$$= \frac{g\rho x \left(\dfrac{AA'}{A' - A}\right)}{V\rho}$$

$$= g\,\frac{AA'}{V(A' - A)} \cdot x.$$

Prop. 11. *To investigate the conditions that the equilibrium
of a floating body may be* stable, unstable, *or* neutral.

Let the figure 12 represent
a section of the floating body
through its center of gravity G.
Let AB be the surface of the
fluid, and H the center of gravity
of the fluid displaced before the
angular disturbance, and when
the line HG was vertical. Let
H' be the center of gravity of
the fluid displaced after the dis-

Fig. 12.

turbance, as in the figure. Draw $H'M$ a vertical line meeting
the line HG produced in M, then M is the *metacenter* of the
floating body.

2—2
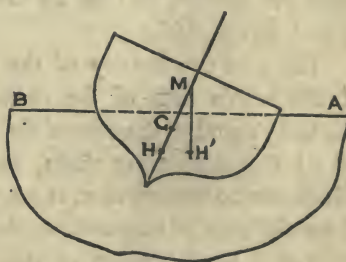
Now if, as in fig. 12, G is below M, there is a statical couple bringing the body back to its first position with the line MGH vertical, the equal and parallel forces being the weight of the body acting vertically downwards through G, and the fluid pressure acting vertically upwards through H'.

If, as in fig. 13, the center of gravity G of the floating body were above M, the moment of the couple would evidently tend to turn the body *from* the original position of equilibrium, which would be therefore unstable.

If M and G coincided, the equilibrium would exist in the new position, and therefore would be neutral.

In stable equilibrium then, the center of gravity of the body is below the metacenter.

In unstable equilibrium the center of gravity is above the metacenter.

Fig. 13.

In neutral equilibrium the center of gravity is at the metacenter.
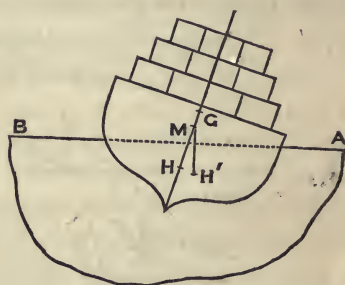
PROP. 12. *To find the* whole *pressure of a fluid upon the surface of a body immersed in it.*

Let $BPCQ$ be the body, PQ an indefinitely narrow horizontal ring upon its surface, and let its area be a, and depth $AM = z$.

Let ρ be the density of the fluid, A the whole area of the surface of the body equal to the sum of the areas of all the elementary rings, as PQ, which can be formed upon it, or, using Σ for the sign of summation, $A = \Sigma(a)$.

Fig. 14.

Let $\bar{z}$ be the depth of the center of gravity of the surface A below the surface of the fluid, and

$$\therefore \bar{z}.A = \Sigma(a.z)$$

by the property of the center of gravity.
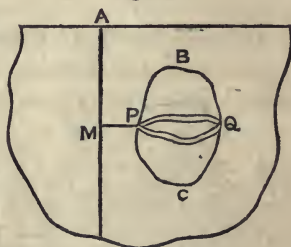
Now the pressure of the fluid upon the ring $PQ = g\rho az$, by Prop. 3, and the whole pressure upon the surface of the body

$$= \Sigma \, (g\rho az)$$
$$= g\rho \Sigma \, (a.z)$$
$$= g\rho A\bar{z},$$

which is the same as if the whole surface of the body were horizontal in the fluid, and at the depth of *its center of gravity*. It will be found in the examples that this proposition is of frequent application to find the pressures on given surfaces in fluids.

COR. If the surface on which the pressure is required be the whole or part of the surface of a vessel containing fluid, the same rule will evidently hold good.

PROP. 13. *To find the* center of pressure *of a plane area immersed in a fluid.*

DEF. By the *center of pressure* we mean the point where the *whole pressure* on the plane surface may be considered to act, and would produce the same mechanical effect as the actual pressures on the surface.

The center of pressure is found in a similar manner to the center of gravity in statics; namely, by taking moments about any given lines. The cases of different forms of floodgates are easily solved by the Integral Calculus, but the case of a rectangular floodgate is of so much practical importance that it will be here solved by Algebra alone.

Let $ABCD$ be the rectangular floodgate, with the side AB in the surface, and inclined at an angle θ to the surface of the fluid, and seen obliquely in fig. 15. Draw OM bisecting the opposite sides AB and CD, then the center of gravity and the center of pressure will both be in the line OM. Take PpQ an indefinitely narrow elementary area parallel to AB, and draw from

Fig. 15.
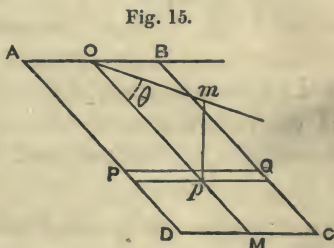
its middle point p the vertical line pm to the surface of the fluid. Join O and m, then the line Om in the surface of the fluid is perpendicular to AB, and the angle $pOm = \theta$ is the angle between the rectangle $ABCD$ and the surface of the fluid; also the depth $pm = Op \sin \theta$.

Let $AB = a$, $BC = b$, and let the breadth of $PQ = \dfrac{b}{n}$, where n is a very large number. Then the area of the element

$$PpQ = a \cdot \frac{b}{n}.$$

The pressure on PpQ

$$= g\rho \times \text{area} \times \text{depth } pm$$

$$= g\rho a \cdot \frac{b}{n} \cdot Op \sin \theta.$$

Now let $Op = m \cdot \dfrac{b}{n}$, where the extreme values of the number m are 0 and n;

$\therefore$ the pressure on $PpQ = g\rho ab^2 \cdot \dfrac{m}{n^2} \cdot \sin \theta,$

and the moment of this pressure about the line AB is

$$\text{pressure} \times \text{arm } Op = g\rho ab^3 \cdot \frac{m^2}{n^3} \cdot \sin \theta.$$

Let X be the distance of the center of pressure from O, also by Prop. 12 the whole pressure upon $ABCD = g\rho \cdot$ area $\times$ depth of center of gravity

$$= g\rho ab \cdot \frac{b}{2} \cdot \sin \theta,$$

and its moment about AB with the arm $X =$ sum of the moments of the pressures upon all the elements such as PpQ about the same line; or, using again Σ for the sign of summation, we have

$$g\rho \frac{ab^2}{2} \cdot \sin \theta \times X = \Sigma \left\{ g\rho ab^3 \cdot \frac{m^2}{n^3} \cdot \sin \theta \right\}$$

$$= g\rho ab^3 \sin \theta \, \Sigma \left(\frac{m^2}{n^3} \right);$$

$$\therefore X = 2b\Sigma\left(\frac{m^2}{n^3}\right),$$

where m is to have every integral value from 0 to n, or

$$\Sigma\,(m^2) = 0 + 1^2 + 2^2 + 3^2 + \&c.\ldots n^2$$

$$= \alpha + \beta n + \gamma n^2 + \delta n^3 \ldots \text{say,} \quad \ast$$

where α, β, γ, δ are constants independent of the number of the terms of the series, and $\alpha = 0$, since $\Sigma\,(m^2) = 0$ when $n = 0$.

Again, carrying the series one term further,

$$0 + 1^2 + 2^2 + 3^2 + \&c.\ldots n^2 + (n+1)^2$$

$$= \alpha + \beta \,.\, (n+1) + \gamma \,.\, (n+1)^2 + \delta \,.\, (n+1)^3;$$

subtracting the former from this,

$$(n+1)^2 = n^2 + 2n + 1 = \beta + \gamma\,(2n+1) + \delta\,(3n^2 + 3n + 1),$$

$$\text{or } n^2\,(3\delta - 1) + n\,(3\delta + 2\gamma - 2) + (\delta + \gamma + \beta - 1) = 0,$$

which is to be true for all values of n, and the coefficient of each power is therefore to be zero,

$$\text{or} \qquad 3\delta - 1 = 0, \text{ and } \delta = \frac{1}{3},$$

$$3\delta + 2\gamma - 2 = 0, \;\ldots\; \gamma = \frac{1}{2},$$

$$\delta + \gamma + \beta - 1 = 0, \;\ldots\; \beta = \frac{1}{6},$$

$$\text{and } \Sigma\left(\frac{m^2}{n^3}\right) = \frac{\frac{n}{6} + \frac{n^2}{2} + \frac{n^3}{3}}{n^3} = \frac{1}{6n^2} + \frac{1}{2n} + \frac{1}{3};$$

and when n is indefinitely great, this sum $= \frac{1}{3}$;

$$\therefore X = \tfrac{2}{3}b,$$

and the center of pressure is below the center of gravity and independent of θ.

If $\theta = 0$, and the surface $ABCD$ is parallel to the surface of the fluid at a given depth, then the center of pressure coincides with the center of gravity, since the pressures are a system of

parallel forces proportional to the areas upon the surface of the floodgate.

The point found as the center of pressure is that where a single force will balance the actual pressures, and it requires particular attention in the construction of vats, tanks, and reservoirs of wood or metal to contain fluids.

If a single force is to be applied to the gates of a dock or canal lock, which shall balance the fluid pressures and relieve the hinges of all strain, it is clear that the center of pressure is the point where it must be applied. If, again, the equivalent force at any point of the gate is required, the moments of the force, and of the whole pressure acting at the center of pressure about the hinges, must be equated to each other.

PROP. 14. *To find the pressure at a given depth when several fluids which do not mix are in equilibrium.*

When a number of fluids which do not mix are placed in any vessel, then when in equilibrium they will arrange themselves in their order of densities; those which are specifically lighter taking the higher positions, and their common surfaces where two meet must be horizontal, in order that the horizontal pressures may balance.

Fig. 16.

The unit of pressure at any point, such as P, fig. 16, is that due to the different fluids in the column above it.
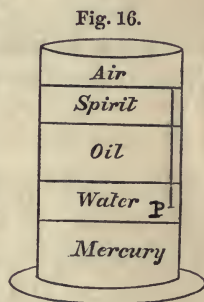
Let the depth of P be

made up of a depth z_1 in a fluid whose density is ρ_1,

.... z_2 .... ρ_2,

.... z_3 .... ρ_3,

.... &c. .... &c.

Then if p_1 is the pressure due to the upper stratum of fluid,

p_2 .... second ....

p_3 .... third ....

&c. .... &c. ....

and if p is the unit of pressure at P or at the depth $z_1 + z_2 + z_3 +$ &c., we have

$$p = p_1 + p_2 + p_3 + \text{\&c.}$$
$$= g\left(\rho_1 z_1 + \rho_2 z_2 + \rho_3 z_3 + \text{\&c.}\right),$$

which result may be arrived at directly, by supposing a cylindrical vertical column to be separated by an imaginary rigid film from the rest, as in Prop. 3, and the sum of the weights of the various portions of the column makes up the weight at the base.

COR. The previous propositions suppose the existence of a vacuum around the fluid which is under consideration, since the fluid pressure was taken to arise from the fluid itself only. When the circumstances occur in the atmosphere, there are cases where the atmospheric pressure requires to be taken into account, whilst in others, being equal in all directions, it does not affect the problem under discussion. The equivalent of the atmospheric pressure, in terms of that arising from any given fluid, will be found further on.

PROP. 15. *To find the conditions of equilibrium when two fluids which do not mix meet in an inverted bent tube.*

The heavier fluid will occupy the lowest portion of the bent tube, as in fig. 17, say from A to D; and the lighter one the other leg, say from A to B.

Fig. 17.

From A draw a horizontal line to C, and let the *vertical* height of D above the level of AC be h, and ρ the density of the fluid; let h' be the height of B above AC, and ρ' the density of the fluid in the leg AB. Then the portion of the fluid from A to C will balance of itself, and the pressure at A must equal that at C, or we must have

$$g\rho h = g\rho' h',$$

and therefore $\dfrac{h}{h'} = \dfrac{\rho'}{\rho}$.
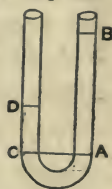
This result holds equally under the pressure of the atmosphere and in a vacuum, since the pressure of the atmosphere

will be the same at B as at D, if we neglect the small difference of their heights.

COR. 1.   The same result evidently holds good whatever be the inclinations of the two legs to the horizon; and also if two vessels of any form were connected by a pipe at their lower ends.

COR. 2.   If each of the tubes or vessels contained several fluids which did not mix, the lowest portion would be occupied by the heaviest; and drawing a level surface AC within it, the condition of equilibrium will be that the whole pressures at that level must be equal.

EXAMPLES.

EX. 1.   A cylinder with its axis vertical being filled with water, find the pressure on the base and concave surface when the height is 10 feet, and the diameter of the base is 2 feet.

ANS.   The weight of a cubic foot of water being 1000 ounces, the pressure on the base is the weight of 31·416 cubic feet of water, and equal to 1963·5 lbs.   By Prop. 12, the pressure on the concave surface equals the weight of 314·16 cubic feet of water equal to 19635 lbs.; since the surface of the cylinder $= 10 \times 2 \times 3·1416$ square feet, and its center of gravity is at the middle point of the axis.

EX. 2.   Show that if the height of the cylinder in the last example were 1 foot in place of 10 feet, the pressure on the base would equal that upon the concave surface.

EX. 3.   Show that a cube with its base horizontal being filled with fluid, the pressure on the base is twice that on any one of the vertical faces.

EX. 4.   Show that if a rectangular parallelopiped, of which the edges are a, b, and c inches, be set with that one of its faces whose sides are b and c inches, horizontal, and be filled with fluid, then the pressures on two contiguous vertical faces will be

in the ratio of b to c; also the pressure on the base equals that on one of the vertical faces multiplied by $\dfrac{2c}{a}$, and equals that upon the other multiplied by $\dfrac{2b}{a}$.

Ex. 5.   The area of the surface of a sphere being that of four great circles, show that if a sphere of one foot diameter be immersed in water with its center 10 feet below the surface, then the whole pressure upon its surface will be 1963·5 pounds.

Ex. 6.   To find the weights upon the upper board of the hydrostatic bellows when the surface of the water in the pipe is at a given height above the upper board of the bellows.

This instrument may be made in a variety of forms.   Before the India-rubber cloth was invented it was frequently made like fig. 18, where AB is a vertical pipe from the center of the upper circular board EF.   The board EF was connected by leather, strengthened by horizontal metal rings, with the lower board CD.   The two boards CD and EF might be brought near together with the metal rings only between them, or be separated to a considerable distance.

Fig. 18.
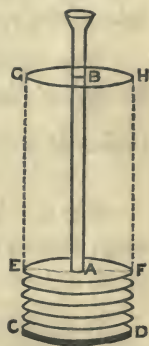
The usual method of now making the instrument is to have the pipe AB at the side, and connected with a pipe leading to the center of the lower board. At the end of that pipe, in the center of the lower board, is screwed the brass fitting in the neck of a bottle of India-rubber cloth, of a cylindrical form, strengthened internally with horizontal metal rings.   The upper board EF then only rests upon the circular end of the bottle.

Now if the bellows and pipe contained water to the height B, it would raise, if free to do so, the upper board EF to the position GBH, since fluids '*tend to find their level;*' and if the upper board is in equilibrium in the position EF, weights must be placed upon it equal to the weight of the column of fluid $EFGH$, which can therefore be easily calculated when the diameter of EF and the height AB are known.

Let the height AB be ten feet, and the diameter of the upper board EF be two feet; then the weights put upon the upper board, together with the board itself, the pipe AB and fluid in it, must equal 1963·5 pounds, the calculation being that of Example 1.

If the diameter of the upper board is one foot, and the height AB is five feet, the weight supported is $245\frac{7}{16}$ pounds.

The effect being independent of the diameter of the pipe, this experiment illustrates the great effect sometimes produced in nature by thin high columns of water which open at their lower ends into close cisterns of moderate dimensions.

Ex. 7. A surface of one square inch being immersed in mercury at the depth of 30 inches, required the pressure upon it, taking the specific gravity of mercury = 13·58.

The pressure will be evidently, by Prop. 3, the weight of 30 cubic inches of mercury = 13·58 × the weight of 30 cubic inches of water = $13\cdot58 \times \dfrac{1000}{1728} \times 30$ ounces = 14·735 pounds.

Ex. 8. What will be the height of a column of water which will produce the same pressure on any given area with a column of 30 inches of mercury?

If we put ρ = the density of water, ρ' = the density of mercury, z and z' the respective depths, the pressure on any horizontal area A being, by Prop. 3, $= g\rho z \,.\, A = g\rho'z' \,.\, A$;

$\therefore\ z = \dfrac{\rho'}{\rho}\,.\,z'$, and by the question $\dfrac{\rho'}{\rho} = 13\cdot58,\ z' = 30$ inches;

and the height of the column of water required $= 30 \times 13\cdot58$ inches = 33·95 feet.

Ex. 9. To find the height of the column of oil, specific gravity ·92, which shall produce the same pressure as a column of 30 inches of mercury.

The height of the column of oil required

$$= 30 \times \frac{13\cdot58}{\cdot92} \text{ inches} = 36\cdot9 \text{ feet.}$$

Ex. 10. A cone, with its base horizontal, being filled with fluid, find the ratio of the height to the radius of the base, when the pressure on the base equals that on the concave surface.

The center of gravity of the concave *surface* of the cone, as found in statics, is in the line drawn from the vertex of the cone to the center of the base, and at $\frac{2}{3}$rds that distance from the vertex.

The concave surface will develope or spread out into a circular sector, of which the arc is the circumference of the base, and the radius is the generating line AB (fig. 19) of the surface (called also the slant side).

Let $r = BC =$ radius of the base,

Fig. 19.

$h = AC =$ height of the cone,

G the center of gravity of the concave surface, and

$AG = \frac{2}{3} h$; also $AB = \sqrt{r^2 + h^2}$.

Then by Prop. 12, the pressure on the concave surface

$$= g\rho\pi r \sqrt{r^2 + h^2} \times \frac{2}{3} h,$$

and the pressure on the base

$$= g\rho\pi r^2 . h,$$

and when these are equal we have

$$\frac{2}{3} \sqrt{r^2 + h^2} = r,$$

or

$$h = r . \frac{\sqrt{5}}{2} .$$
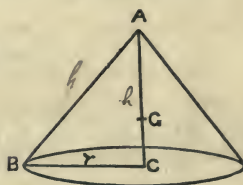
Ex. 11. Show that in the regular tetrahedron filled with fluid, and one face for the base horizontal, the pressure on each of the inclined faces equals two-thirds that on the base.

Ex. 12. If a cone resting on its base be filled with fluid, show that when the height is three times the radius of the base,

the pressure on the concave surface is (2·108 times) more than twice that on the base.

Ex. 13. A cylinder, of which the density is ρ', floats in equilibrium at the surface of a fluid whose density is ρ, with its axis vertical; to find how deep it will sink. Also the density of ice being $\frac{8}{9}$ that of water, to show that the thickness of a sheet of ice floating at the surface of water equals nine times the height of its surface above the water.

Let CMD be the level of the surface in fig. 20, and let $AM = x$, to be found when $AB = h$ is given.

Fig. 20.

From Prop. 8, $\quad \dfrac{V}{V'} = \dfrac{\rho'}{\rho} = \dfrac{x}{h}$;

$$\therefore \ x = h \cdot \frac{\rho'}{\rho};$$

also $BM = h\left(\dfrac{\rho - \rho'}{\rho}\right)$, which gives h when BM is known.
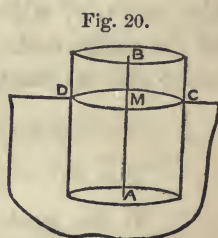
Ex. 14. A cone floats in a fluid with its axis vertical and apex downwards; to find how deep it will sink.

Let $AB = h$ be given, and $AM = x$ to be found, in fig. 21. Since the plane of floatation cuts from the cone a portion, as in the figure, similar to the whole cone, we have

Fig. 21.

$$\frac{V}{V'} = \frac{x^3}{h^3} = \frac{\rho'}{\rho};$$

$$\therefore \ x = h \cdot \sqrt[3]{\frac{\rho'}{\rho}}.$$
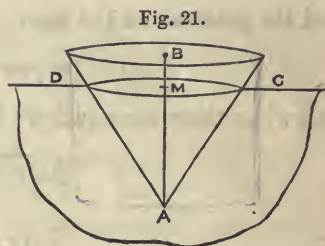
Ex. 15. To find how deep a paraboloid of revolution will sink in a given fluid, the volume of a segment of a paraboloid being one-half that of a cylinder of the same base and altitude.

Let $AB = h$, $AM = x$, DME being in the surface of the fluid in fig. 22, and

Fig. 22.

$$DM^2 = 4mx, \quad BC^2 = 4mh.$$

Then the volume of the whole parabolic segment

$$= \tfrac{1}{2}\pi \cdot BC^2 \times AB = \tfrac{1}{2}\pi \cdot 4mh^2,$$

and the volume of the part immersed

$$= \tfrac{1}{2}\pi DM^2 \times AM = 2\pi mx^2,$$

and by the formula of Prop. 8,

$$\frac{V}{V'} = \frac{\rho'}{\rho} = \frac{x^2}{h^2},$$

and

$$x = h\sqrt{\frac{\rho'}{\rho}}.$$
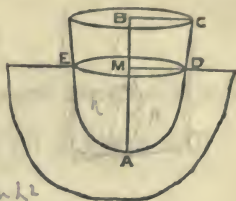
Ex. 16.   To find how deep a cylinder floating in a fluid with its axis horizontal will sink.

The depth to which it will sink will be independent of the length of the cylinder.

Let AOB be the vertical diameter, and O the center of the perpendicular section of the cylinder in fig. 23, and DC the surface of the fluid; also let the angle $AOC = \theta$, the radius $AO = a$, and

Fig. 23.

$$AM = x = a \text{ vers } \theta,$$

the area of the segment $ACD =$ area of sector $ACOD -$ area of triangle DOC

$$= \tfrac{1}{2} \text{arc} \times \text{rad.} - \tfrac{1}{2}DC \times OM = a^2 \cdot \theta - \tfrac{1}{2}(2a\sin\theta \cdot a\cos\theta)$$

$$= a^2(\theta - \tfrac{1}{2}\sin 2\theta),$$
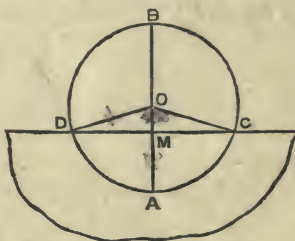
and

$$\frac{V}{V'} = \frac{\rho'}{\rho} = \frac{\text{area of segment}}{\text{area of circle}} = \frac{\theta - \tfrac{1}{2}\sin 2\theta}{\pi},$$

and if θ is found from the expression

$$\theta - \tfrac{1}{2}\sin 2\theta = \pi \cdot \frac{\rho'}{\rho},$$

then x is known from $x = a$ vers θ.

The equation for finding θ is

$$\theta + \tfrac{1}{2}\sin 2\theta = \pi \cdot \frac{\rho'}{\rho},$$

when the axis of the cylinder is below the surface of the fluid.

Ex. 17. If a vat, or a barrel of a cylindrical form, with its axis vertical, the bottom closed and the top open, had the staves parallelograms, where could a single hoop be placed to counteract the fluid pressure upon the staves when it was filled with fluid?

Ex. 18. Suppose a single hoop were placed in the last experiment *higher* than the center of pressure, what would be the effect?

Ex. 19. Suppose the hoop were placed *lower*, what would be the effect?

Ex. 20. Suppose the hoop placed in any given position, and the vat were only partly filled with fluid, what would be the result?

CHAPTER II.

ON HYDROSTATICAL INSTRUMENTS.

PROP. 16. *To explain the construction and principle of the common hydrometer.*

The common hydrometer *ABME* consists of two bulbs *A* and *B* blown upon a tube of glass with the blowpipe. The bulb *A* is weighted to cause the instrument to float with the stem *ME* vertical, and the amount of weight in *A* is regulated for the fluid to which it is to be applied; its sensitiveness is increased as the diameter of the stem is diminished with the same dimensions of bulb.

Fig. 24.

A slip of paper being introduced at the end *E* before it is closed, the instrument is placed in fluids of known but different specific gravities, and the depth to which it sinks in each is noted. These and intermediate divisions being marked upon the slip of paper, it is put into its place in the ,tube and the end *E* closed with the blowpipe.

When the hydrometer is placed in any liquid and floats in equilibrium, it displaces its own weight of the liquid; and the volume displaced will be less as its density and specific gravity are greater, and this is indicated by the depth to which it sinks, as at the point *M* in the surface *CMD* of the liquid, and this is easily read off.

Being made of glass, it has the advantage, besides its cheapness, that it may be placed in acid and saline solutions without injury.
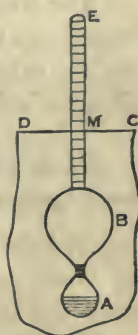
Prop. 17.   *To explain the construction and uses of Sikes's hydrometer.*

Fig. 25.

Sikes's hydrometer is made of brass similar to fig. 25, having a solid ball A at the lower end, connected by a tapering stem with a hollow ball B; from B is the flat brass rod BC on which the divisions are marked. From the smallness of the rod BC the instrument is very sensitive, and at the same time a considerable range is obtained by its being provided with a set of brass weights, such as D, which will pass over the narrow upper part of the stem between A and B, and rest upon the ball A.
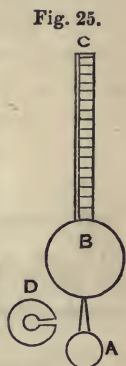
The way in which the specific gravity of liquids is found by Sikes's hydrometer, is like that of the common hydrometer, but the value of the divisions for the different weights, such as D, must be known.

Prop. 18.   *To explain the construction and uses of Nicholson's hydrometer.*

Nicholson's hydrometer is chiefly made of brass, but of different forms. When made like fig. 26, it has a cup at the lower part A, in which bodies can be placed for the purpose of weighing them in water. This cup is connected with the hollow ball B, from the upper part of which rises a steel wire, and on the end of it is the support for the cup D. When in use the instrument is sunk in the liquid to the mark

Fig. 26.   Fig. 27.

C upon the steel wire in fig. 26, or H in fig. 27. When made of the form fig. 27, it has a solid ball at E, a hollow vessel with a flat top at F, and the steel wire with mark H rises from a bridge of brass wire abc. The cup G is supported on the end of the steel wire as before.
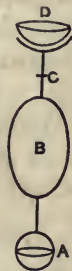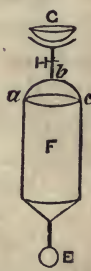
The instrument is furnished with sets of weights by means of which it is sunk to the marks C or H in the fluid. With

1000 of the grain or half-grain weights in the upper cup, it will float in distilled water with the mark C or H in the surface. If any body of less than that weight be placed in the cup, the weights which are required in addition to sink it again to the same mark, being taken from 1000, we have the difference, the weight of the body in air. If the body is placed in the cup A, or on the flat top ac, and the same process is followed, the weight of the body in water is found, and then its specific gravity can be calculated as with the hydrostatic balance.

From the smallness of the steel wire on which the marks C and H are made, the instrument is exceedingly sensitive, and will serve instead of a delicate balance; but it is very troublesome, being affected by the varying temperature of the water with which it is used.

To find the specific gravity of liquids by Nicholson's hydrometer the instrument itself must be weighed. Then this weight, together with the weights to be put in the cups D or G to sink the instrument to the marks C or H, gives the weight of the fluid displaced, and thus the weights of the same bulks of distilled water and any other liquid can be compared.

Various forms of the hydrometer have had the names *areometer*, *alcoholmeter*, *saccharometer*, &c.

PROP. 19. *To explain the uses of the specific gravity bottle.*

The specific gravity bottle is made like fig. 28.

Fig. 28.

It has a stopper AB made of a piece of glass tube, which is ground into the neck of the bottle until the distilled water, which will fill the bottle and the pipe of the stopper, weighs exactly 1000 grains.

If the bottle be then filled with any other liquid, its specific gravity is known at once to great accuracy, by weighing again, the volume being that of 1000 grains of distilled water.

The specific gravity bottle is also of use to determine the specific gravities of powders, sand, and small crystals, &c., not

3—2
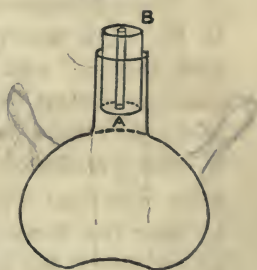
soluble in water; for when a given weight of any of these is put into the bottle, and then the remaining space filled with distilled water, and the bottle then weighed, we have the weight of distilled water of equal bulk to the bodies from the following formula.

Let W be the weight of the bottle when holding 1000 grains of water,

w the weight of powder, sand, &c. introduced into it,

W' the weight of the bottle when filled with sand, &c. and water.

Then the weight of distilled water equal in bulk to the powder, sand, &c.

$$= W + w - W',$$

and the specific gravity of the powder, sand, &c.

$$= \frac{\text{the weight of the sand, \&c. in air}}{\text{the weight of an equal volume of water}}$$

$$= \frac{w}{W + w - W'}.$$

PROP. 20.   *To explain the uses of the hydrostatic balance.*

The hydrostatic balance, as in fig. 29, is merely the common balance, with one of the scale-pans hung with shorter cords for the convenience of weighing bodies in water when hung from it, as A in the figure, by some fine thread, or best of all, by a very fine hair.

Fig. 29.
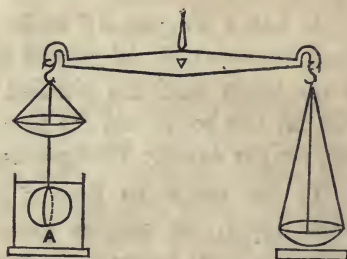
Let w be the weight of the body in air,

w' .... water.

Then the weight of the water displaced by the body equals the weight lost by weighing in water, by Prop. 6,

$$= w - w',$$

and the specific gravity of the body

$$= \frac{\text{weight of the body}}{\text{weight of an equal volume of water}}$$

$$= \frac{w}{w - w'},$$

where the specific gravity of water is taken unity.

In the above discussion it is supposed that the body sinks in water. If the body is specifically lighter than water, it is necessary to hang a heavy body to it to cause it to sink.

Let $w = $ the weight of the light body in air,

$w_{,} = $ .... heavy ....

$w' = $ .... heavy body in water,

$w'' = $ .... both together..........

The weight of both in air minus the weight of both in water equals the weight of water displaced by both

$$= w + w_{,} - w'',$$

the weight of water displaced by the heavier

$$= w_{,} - w';$$

therefore by subtracting, the weight of water displaced by the lighter body

$$= w + w' - w'',$$

and the specific gravity of the light body

$$= \frac{w}{w + w' - w''}.$$

The specific gravity of a liquid is easily found with the hydrostatic balance by weighing any heavy body in air, in water, and in the liquid; since the weight lost by weighing in water equals the weight of the water displaced, and the weight

lost by weighing in the liquid equals the weight of the liquid displaced; and the volumes displaced are equal.

Let w = weight of the body in air,

w' = ... water,

w'' = ... the liquid.

Then the specific gravity of the liquid $= \dfrac{w - w''}{w - w'}$.

The specific gravity of a body is one of the characters which enable us in many cases to determine its nature. In mineralogy, the form, crystalline or otherwise, the form of fracture, the lustre, the colour, the hardness, the colour of the streak when soft, and specific gravity are generally sufficient to determine the mineral, if it is of a known species. The simple metals are readily distinguished by their appearance and specific gravity, and the composition of the alloys may be often known approximately by the same means.

If the components in an alloy of two metals, or a mixture of two liquids, are known, the proportions of each can be found by taking the specific gravity of the alloy or mixture, and allowing for the change of bulk which generally takes place on the combination of liquids, or of metals in a state of fusion. The alloys of some metals occupy less bulk than their components did, and those of others occupy more space. The taking the specific gravity of the alloy or mixture, when the proportions are known, enables us to ascertain whether there has been expansion of volume or condensation during the combination.

The true weight of a body is its weight in a vacuum, its apparent weight in air is its true weight minus the weight of an equal bulk of air, since air acts like other fluids, producing a resultant vertical pressure equal to the weight of the air displaced. The air continually changing in density, when great nicety of weighing of large bodies is attempted, the nature of the weights, as well as of the body weighed, require to be considered.

PROP. 21. *To find the specific gravity of an alloy or mixture of given composition, supposing no change of bulk to have arisen.*

Let ρ_1 be the density of one of the components, V_1 its volume, and w_1 its weight,

ρ_2 the density of the other component, V_2 its volume, and w_2 its weight,

ρ the density of the alloy or mixture, V its volume, and w its weight.

Then $V = V_1 + V_2$, and $w = w_1 + w_2$

$$= g\rho V;$$

$$\therefore \rho = \frac{w_1 + w_2}{gV}$$

$$= \frac{w_1 + w_2}{g(V_1 + V_2)} = \frac{w_1 + w_2}{\dfrac{w_1}{\rho_1} + \dfrac{w_2}{\rho_2}}$$

$$= \frac{(w_1 + w_2)\,\rho_1\rho_2}{w_1\rho_2 + w_2\rho_1},$$

which gives ρ when the weights and densities of the components are known, and if by experiment it differs from this we know whether they have contracted or expanded on being combined.

PROP. 22. *An alloy or mixture having undergone condensation, to find the degree of condensation from the known specific gravities.*

Since the mass is unchanged, let V and ρ be as in the last proposition, V' and ρ' the actual volume and density of the mixture or alloy, then $V\rho = V'\rho'$,

$$\text{and } V - V' = V\left(1 - \frac{\rho}{\rho'}\right);$$

therefore the condensation $= \dfrac{V - V'}{V} = \dfrac{\rho' - \rho}{\rho'}$.

Cor.   Similarly for expansion,

the expansion $= \dfrac{V' - V}{V} = \dfrac{\rho - \rho'}{\rho'}.$

Prop. 23.   *To find the proportions of the components in an alloy of given metals, supposing no change of volume to have taken place, when the specific gravity or density is known.*

Let $\rho_1 V_1 w_1$; $\rho_2 V_2 w_2$, $\rho V w$ be as in Prop. 21, and now w_1 and w_2 are to be found, supposing that $V = V_1 + V_2$;

but $w = w_1 + w_2$

$$= w_1 + g\rho_2 V_2$$

$$= w_1 + g\rho_2 (V - V_1)$$

$$= w_1 + g\rho_2 \left(\frac{w}{g\rho} - \frac{w_1}{g\rho_1} \right);$$

$$\therefore w_1 \left(1 - \frac{\rho_2}{\rho_1} \right) = w \left(1 - \frac{\rho_2}{\rho} \right),$$

or $w_1 = w \dfrac{\rho_1}{\rho} \left(\dfrac{\rho - \rho_2}{\rho_1 - \rho_2} \right),$

and $w_2 = w - w_1 = w \cdot \dfrac{\rho_2}{\rho} \left(\dfrac{\rho_1 - \rho}{\rho_1 - \rho_2} \right).$

Cor.   These values of w_1 and w_2 will be only approximations, on account of the condensation or expansion, but would serve to find the true values more nearly if the law of condensation or expansion, depending upon the proportion $\dfrac{w_2}{w_1}$, were known.   Thus let m be a given number, and

$$mV = V_1 + V_2,$$

and proceeding as before,

$$w_1 = w \frac{\rho_1}{\rho} \left(\frac{\rho - m\rho_2}{\rho_1 - \rho_2} \right).$$

These formulæ apply in like manner to the mixtures of liquids.

PROP. 24. *To explain the construction and uses of the spirit-level.*

The plumb-line hanging always perpendicular to the surface of still water, it was formerly the general means of determining the horizontal and vertical directions.

The first fluid-levels were formed like fig. 30, by a tube bent twice at right angles, and filled partly with fluid, as in *ABC*. Now *A* and *C* being the

Fig. 30.

free surfaces in the same level, an eye *E* looking from *C* past *A* will see the objects in the same level with them.
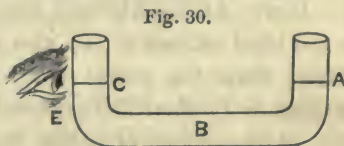
This method is not very accurate, and even when floats with sights were applied, it was only applicable to very ordinary purposes.

The spirit-level, as now applied to astronomical and surveying instruments, is susceptible of very great accuracy. The essential part

Fig. 31.

consists of a glass tube, as *AB*, fig. 31, slightly curved, and containing spirit of wine except a bubble at *a* in the figure. The ends being closed with the blowpipe, the spirit and bubble remain the same, and the bubble only rests at the highest part. This instrument is more sensitive, as the tube is more nearly straight, for then a small change of inclination to the horizon causes the bubble to move through a larger space.
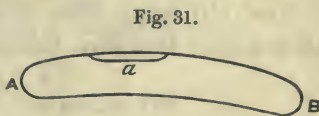
When the tube, fig. 31, mounted in an appropriate brass tube with means of adjustment, is attached to a telescope, they constitute the telescopic level, which is of continual use in surveying for many purposes, as drainage, the laying out of canals, railways, &c. A level surface in Prop. 3, Cor. 3, is defined to be that in which water or any other liquid rests, and, when of small extent, is a horizontal plane; but when larger, such as the still surface of a lake or the sea, it has the curvature of the earth's surface. This curvature has to be considered and allowed for in such operations as levelling for carrying a canal over a country, to determine the gradients of railways, &c.

PROP. 25. *To investigate an expression for the depression in levelling.*

We may consider the earth as a sphere of 3956 miles radius. Let ADE represent in fig. 32 the level surface through A and D. Draw AB a horizontal line and tangent to the level surface at A; also C being the center, draw the secant CDB meeting the tangent in B, then BD is called the depression of the point D below the horizontal line AB.

Fig. 32.
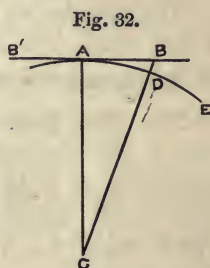
By Euclid, Prop. 36, Book 3, $BD(BC + CD) = AB^2$; and since BD is small compared with CD, we have

$$BD = \frac{AB^2}{2 \cdot CD}, \text{ nearly.}$$

Now CD being 3956 miles, the formula expresses BD and AB in miles; therefore, bringing BD into feet, we have

$$BD \text{ in feet} = \frac{(AB \text{ miles})^2 \times 3 \times 1760}{2 \times 3956}$$

$$= \frac{2}{3}(AB \text{ miles})^2, \text{ nearly;}$$

or BD in inches $= 8$ inches $\times (AB \text{ miles})^2$;

or if AB is 1 mile, then BD is   8 inches,

...... AB is 2 .... BD is 32 ....

...... AB is 3 .... BD is 72 ....

&c.   &c.

We see that the depression soon amounts to a large quantity on even the lake and canal surfaces which are met with in this country. In actual surveys, the atmospheric refraction requires to be taken into account in reducing the observations.

COR. If B were the place of the eye of an observer, then A is called *the offing* or visible horizon; and if BD is given, by the formula

$$AB \text{ miles} = \sqrt{\frac{3}{2} BD \text{ feet}}.$$

If BD is 6 feet, then AB is 3 miles; and if BD is 24 feet, then AB is 6 miles, &c. If B' is a point seen from B beyond the offing A, we have $BB' = AB + AB'$, which is found by the above rule, if the depressions at B and B' are given.

PROP. 26. *To find the mechanical advantage of Bramah's press.*

Bramah's press consists essentially of a powerful forcing pump connected by a pipe with a strong hollow cast iron cylinder, in which works, water-tight, a solid cylinder or ram of the same material. The hydrostatical principles in effect in the press are, that fluids transmit pressure equally in all directions; and that the pressures, on any plane areas within them, are proportional to the areas.

Let ABC be the lever handle of the forcing-pump, and P the power acting at A and in equilibrium on the lever with the reaction Q from the rod of the plunger acting at B, then

$$P \times AC = Q \times BC.$$

Fig. 33.
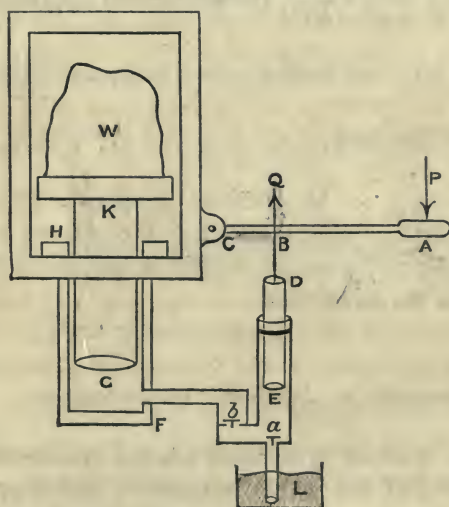
Let DE be the plunger of the forcing-pump working water-tight in the cylindrical barrel of the pump, of which a and b are

strong metal valves opening upwards; and let L be the cistern supplying the water for the press.

Let FH represent the hollow iron cylinder in which the solid ram GK works water-tight. On the stage at K, upon the head of the ram, let the weight W be supported. It is required to find the relation of W and P, or the mechanical advantage $\dfrac{W}{P}$, neglecting the weights of the ram, the lever, and the plunger, which, with a counterpoise on the lever, may balance each other.

Let r be the radius of the ram GK,

$\qquad r'$ .... plunger DE,

$\qquad p$ the unit of pressure of the water, which, neglecting the weight of the valve b, will be the same in the cylinders of the pump and the ram. Then in equilibrium

$$W = p \times \pi r^2,$$
$$Q = p \times \pi r'^2 = P \times \frac{AC}{BC};$$

and dividing, $\dfrac{W}{P} = \dfrac{r^2}{r'^2} \times \dfrac{AC}{BC}$, the mechanical advantage required.

Example, let $r = 6$ inches, $r' = \frac{1}{2}$ inch, and $\dfrac{AC}{BC} = 10$; then

$$\frac{W}{P} = 36 \times 4 \times 10 = 1440;$$

a large advantage to be obtained with only small loss by friction in the leather collars by which the plunger and ram are kept water-tight, and thus, in respect of friction, it has a great advantage over the screw press.

Examples in Hydrostatical Instruments.

Ex. 1. A common hydrometer has the cylindrical stem one fifth the volume of the whole instrument, and floats with the whole stem above the surface in water; what is the specific gravity of the liquid in which it will sink to the top of the stem?

Since the weight of the fluid displaced equals the weight of the instrument when it floats in equilibrium, we have $g\rho V = g\rho' V'$; and from the formula $\frac{V}{V'} = \frac{\rho'}{\rho}$, where $\rho = 1$, the specific gravity required is ·8

Ex. 2. Show that if the hydrometer of the last question be placed in a fluid of the specific gravity ·9, it will float with $\frac{5}{9}$ths the length of the stem above the fluid.

✓ Ex. 3. The specific gravity bottle which contains 1000 grains of water has 150 grains of sand put into it, and then being filled with water it weighs 90 grains more than when filled with water only; show that the specific gravity of the sand is 2·5.

Ex. 4. Equal parts by weight of water and strong sulphuric acid of specific gravity 1·846 being mixed, show that if there had been no change of volume, the specific gravity of the diluted acid would have been 1·2972.

Ex. 5. The specific gravity bottle being filled with the diluted acid of the last question, it is found to weigh 1388·4 grains; show that the condensation in volume has been rather less than one fifteenth part, being $\frac{1}{15·2}$th.

✓ Ex. 6. A piece of a simple metal weighing $113\frac{1}{2}$ grains in air is found to lose 10 grains of weight on being weighed in water; what is the metal?

✓ Ex. 7. A crystal of barytic spar weighs in air 111 grains, and in water 87; show that the specific gravity is 4·625.

✓ Ex. 8. A piece of calcareous spar weighs in air 187 grains, and in water 117 grains; show that the specific gravity is 2·67.

✓ Ex. 9. A piece of mahogany weighs in air 372 grains, a brass weight, hung to it to sink it, weighs 385 grains in water, and the two together weigh in water 302 grains; show that the specific gravity of the mahogany is ·817.

Ex. 10. The specific gravity of pure gold being 19·3, and that of copper 8·9, show that the specific gravity of standard

gold, which consists in every 24 carats of 22 carats gold and 2 carats copper, would be 17·58, if there were no change of volume.

Ex. 11. The specific gravity of standard gold being found by experiment to be 17·157, show that the expansion in volume is rather less than one fortieth part, and is $\frac{1}{40·56}$ th.

Ex. 12. A pebble weighs in air 131 grains, in water 81 grains, and in pyroxilic spirit 88·9 grains; show that the specific gravity of the pyroxilic spirit is ·842, and that of the pebble is 2·62.

Ex. 13. Some plumber's solder, composed of lead and tin, has the specific gravity 8·878; determine approximately the component parts of the solder, the specific gravity of lead being 11·35, and that of tin being 7·29.

Ex. 14. A Nicholson's hydrometer requires 1003 half grains to sink it to the fixed mark in water; when a piece of granite is put in the upper cup it requires 289 to sink it to the fixed mark, and when it is put in the lower cup it requires 556·4; show that the specific gravity of the granite is 2·66.

Ex. 15. A person standing 96 feet above the level of the sea observes a part of the mast of a ship beyond the offing, which is 24 feet above the water line; show that the distance of the ship from the observer is 18 miles.

CHAPTER III.

ON ELASTIC OR AERIFORM FLUIDS.

IT was explained in the Introduction how the fluidity of matter
arose from the caloric, which is an essential part of all bodies as
we know them; and in perfect fluids the molecular attractions
have ceased to produce any sensible effect of aggregation or ad-
hesion. The properties of gases and vapours involve essentially
the consideration of temperature, and can only be fully discussed
when the subject of heat has been studied. It is thus necessary
to take the definition and measure of temperature as known, in
order to discuss even the more prominent properties of elastic
fluids.

By temperature we mean the sensible heat which affects the
thermometer; and the thermometer being an instrument for
measuring temperatures, its scale of degrees is supposed to be
constructed so as to indicate equal increase or decrease of tem-
perature for each change of a degree by the index of the instru-
ment. In the fluid thermometers, as the mercurial, spirit, and
air thermometers, the index of the scale is the free surface of
the fluid in the tube; but in those of solid materials, as Bre-
guet's thermometer, Daniell's pyrometer, &c., it is a pointer
moving along the scale of the instrument. In the investigation
of the ordinary laws of elastic fluids the accuracy of the thermo-
metric scales of degrees may be taken for granted, whilst the
higher parts of the subject will involve the theory of the ther-
mometer itself, to be afterwards discussed.

Again, the experiments being generally performed in the
atmosphere, its fluid pressure is involved in the results, and has
to be considered, together with other pressures of the experi-
ment. We may assume that it is known from the height of the

barometer, of which the theory will be discussed afterwards; and thus can be expressed in terms of the pressure produced by a column of any other given fluid.

The elastic fluids are those which require the constraint of vessels, or external pressures, to keep them at given volumes, and which expand when relieved from the reactions by which they are retained at any given density. The elastic force with which they tend to expand is measured by the pressure it produces upon a unit of area when constant or uniform, and is connected in three primary laws with the volume and temperature of the elastic fluid.

Firstly, the law of Boyle (or Mariotte) for the relation of the elastic force and volume, *when the temperature remains constant.*

Secondly, the law of Gay Lussac for the relation of the volume and temperature, *when the elastic force is constant.*

Thirdly, the law of Amonton, from which the relation of the pressure to the temperature is found, *when the volume remains constant.*

These are to be considered as only approximations, applying with sensible accuracy in ordinary circumstances, but failing in extreme cases.

Law 1.   *When the temperature remains constant the elastic force of a gas is inversely proportional to the volume it occupies.*

For the experimental proof of the law at pressures greater than the atmospheric pressure, let ABC represent, in fig. 34, a bent tube of glass which is closed at A and open at C; then a portion of gas occupying the closed leg from A to a being separated by mercury or other liquid in the lower bend of the tube aBb from the atmosphere, if a and b are in the same level the liquid in aBb is in equilibrium of itself, and the pressure upon the surface at a equals that at b, or the elastic force of the gas in the leg Aa, and acting upon the surface at a, equals the pressure of the atmosphere acting upon the surface

Fig. 34.

a, equals the pressure of the atmosphere acting upon the surface
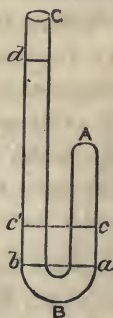
of the liquid at b. Let the pressure of the atmosphere on a unit of area at b be p. Let ρ be the density of the liquid used in the experiment, and h the height of the column of it which will produce by its weight the pressure p, then $p = g\rho h$ by Prop. 3. Now if more liquid be poured into the tube by the open end C, the gas in the closed leg will be found to be compressed into a less space. Let c and d be the free surfaces of the liquid in equilibrium, draw a horizontal line from c to c', the portion cBc' will balance of itself, and the elastic force of the gas in Ac now balances the pressure of the atmosphere on the surface at d, together with that from the column of liquid $c'd$. Let h' be the height of d above the level $c'c$, and the pressure from it is $g\rho h'$. The elastic force of the gas in Ac is now balanced by the pressure p', such that

$$p' = g\rho h + g\rho h' = g\rho\,(h + h').$$

Put $e =$ the elastic force of the gas when it occupied the volume V in Aa; e' the elastic force when it occupies the volume V' in Ac; then it is found for experiments within a very large range, and until a gas comes near its point of liquefaction, that

$$\frac{p}{p'} = \frac{e}{e'} = \frac{h}{h + h'} = \frac{V'}{V},$$

or if p'', e'', V'' were any other corresponding values of p, e, and V, then

$$\frac{p''}{p'} = \frac{e''}{e'} = \frac{V'}{V''};$$

or the elastic force of the gas is inversely proportional to the space it occupies.

Since the quantity of gas is the same if ρ', ρ'' are the densities when the volumes are V' and V'', we have the mass

$$= V'\rho' = V''\rho'',$$

and
$$\frac{V'}{V''} = \frac{\rho''}{\rho'} = \frac{e''}{e'} = \frac{p''}{p'},$$

or the elastic force of a gas varies directly as its density.

Also $p'' = \dfrac{p'}{\rho'}\cdot\rho''$, and if the ratio $\dfrac{p'}{\rho'}$ be given for any one case we may put κ for its value; and now omitting the distinc-

tive marks we have a general expression as the result of Boyle's law, $p = \kappa\rho$.

The value of κ will be found simply expressed further on. For pressures less than that of the atmosphere let BE, fig. 35, represent a vessel containing liquid, and let AD represent a tube within it, closed at A and open at D and filled with the liquid. Let a portion of gas be passed into the tube so as to occupy the space Aa when the level of the surface of the fluid outside is BB', and a the surface inside is in the same level; then the elastic force of the gas acting at a balances the pressure of the atmosphere on the surface BB', since B, a and B are in the same level

Fig. 35.

plane. Let part of the exterior fluid be removed until the surface is at the level $Cc\,C'$, then the gas inside will be found to have expanded to a space as Ad, and if the height $cd = h'$ and p' is now the pressure at d, e' the elastic force of the gas, ρ' the density, and V' the volume, we have

$$p' + g\rho h' = g\rho h.$$

From the result of experiments it is found that as before

$$\frac{p}{p'} = \frac{h}{h - h'} = \frac{e}{e'} = \frac{V'}{V};$$

and the law applies to pressures both greater and less than that of the atmosphere.

Law 2. *When the pressure upon a gas remains the same, the increase or decrease of volume is directly proportional to the increase or decrease of the temperature respectively.*

It was discovered by Gay Lussac and Dalton at nearly the same time, that the gases generally increase from a volume 8 at the freezing point of water to a volume 11 at the boiling point, and the expansion is uniform, under a constant pressure.

Let α be the expansion for each degree of the thermometer, T^0 the number of degrees between the freezing and boiling points. Then
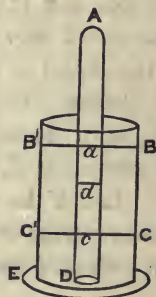
$$\alpha T^0 = \frac{11 - 8}{8} = \cdot 375,$$

and $$\alpha = \frac{\cdot 375}{T^0} \, .$$

The value of α depends upon the number of degrees T upon the scale of the thermometer which is used. If $T^0 = 100^\circ$ as on the centigrade scale, then $\alpha = \cdot 00375$, and if $T^0 = 180^\circ$ as on Fahrenheit's scale, which is used in this country, then

$$\alpha = \frac{3}{8 \times 180} = \frac{1}{480} \, .$$

Let V_0 be the volume of a gas at the freezing temperature when $t^0 = 0$, the volume V at t^0 above that point, then

$$V = V_0 + V_0 \alpha t^0$$
$$= V_0 (1 + \alpha t^0) \, ;$$

or the volumes form an arithmetic progression when the temperatures are in arithmetic progression; this is strictly Gay Lussac's law. Dalton's views will be found discussed in the chapter on Heat. The above law was found by Dulong and Petit to hold with considerable accuracy from nearly the solidifying temperature of mercury to the temperature of boiling water, the temperatures being those shown by the common mercurial thermometer. From the boiling point of water to that of mercury there was a considerable deviation from the law; which was undoubtedly more due to the unequal expansion of the mercury in the thermometer, than to that of the gas. We shall be however far from justified in taking the law as strictly a physical law, or, indeed, as anything more than a most useful empirical law.

With a better method of experimenting it was found by Rudberg that 100 measures of atmospheric air expand to 136·4 or 136·5 between the freezing and boiling points of water, instead of to 137·5, as found by Gay Lussac; this gives

$$\alpha = \frac{1}{494} \, ,$$

which should be now used,

$$\text{and} \quad V = V_0 \left(1 + \frac{t^0}{494}\right),$$

where t^0 are the degrees above 32^0 the freezing point, on Fahrenheit's scale; and to be taken negative below that point.

Again, with very superior methods of experimenting Regnault has found that α is not exactly the same for all gases. He found that between the freezing and boiling points of water

100 measures of hydrogen gas   expand to 136·61

100 .... carbonic acid gas .... 137·10

100 .... sulphurous acid gas............ 139·03

100 .... cyanogen gas   .... 138·77

from which the values of α for these gases must be calculated where great accuracy is required.

Law 3. *The general relations of the pressure, density, and temperature of a gas are given by the formula* $p = \kappa\rho\,(1 + \alpha t^0)$ *obtained by compounding the two previous laws; and therefore when the volume and density are constant the pressure varies as* $(1 + \alpha t^0)$.

Let p_0, ρ_0, V_0 be the commencing pressure, density, and volume when $t^0 = 0$ at the freezing point respectively,

$$\text{then} \quad p_0 = \kappa\rho_0, \text{ by Boyle's law.}$$

Let the density change from ρ_0 to ρ', and the volume from V_0 to V', when the temperature changes from the freezing point to t^0 above it, and the pressure remaining p_0,

$$\text{then} \quad \frac{V'}{V_0} = 1 + \alpha t^0 = \frac{\rho_0}{\rho'}, \text{ by Gay Lussac's law.}$$

Let again the pressure change from p_0 to p, whilst the density changes from ρ' to ρ, and the temperature remains at t^0 above freezing,

$$\text{then} \quad \frac{p}{p_0} = \frac{\rho}{\rho'} = \frac{\rho}{\rho_0}\,(1 + \alpha t^0),$$

or
$$p = \frac{p_0}{\rho_0} \cdot \rho \, (1 + \alpha t^o) = \kappa \rho \, (1 + \alpha t^o),$$

and when ρ or the volume is constant,

$$p \propto (1 + \alpha t^o).$$

For temperatures below the freezing point, t^o must be taken negative.

On the Atmosphere.

The atmosphere acts like other fluids, producing a resultant pressure on a body immersed in it, which equals the weight of the air displaced and acts vertically upwards through the center of gravity of the air displaced. It is found that 100

Fig. 36.

cubic inches of air weigh 31 grains very nearly at the average heights of the barometer and thermometer, and these would be contained in a cube of which the edges were 4·642 inches. We see that this buoyancy from the air may soon become of a magnitude distinctly sensible in ordinary balances and on bodies of moderate dimensions.
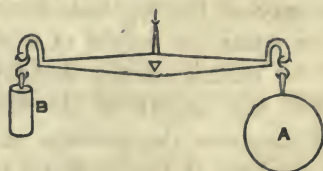
An air-pump experiment, like fig. 36, shows this property of the air very evidently. A closed hollow glass globe as A in the figure, about 3 inches diameter, hanging from one end of the beam of a small balance is counterpoised in air by a metal weight B. When the instrument is placed under the receiver of an air-pump and the air withdrawn, it is found that A and B no longer balance, but that A is the heavier, which becomes more evident as more air is withdrawn from the receiver.

Such a question as—which is the heavier, a pound of iron or a pound of lead?—requires it first to be stated what is meant by the pound weight, and whether the absolute weights

in a vacuum, or the apparent weights in air, are intended, before an answer can be expected.

The *barometer* is an instrument which measures *the pressure* of the atmosphere. In the simplest form it consists of a glass tube AB, 33 or 34 inches long, closed at the end A and open at B, with a cup CD, fig. 37, to hold mercury. The tube AB with the end B upwards, being filled with pure mercury recently boiled to free it from air and moisture, and all air-bubbles being removed from the inside of the tube, the end B being closed with the thumb it is inverted, as in fig. 37, with the end B in the cup DC containing mercury, and then the end B being below the surface the thumb is withdrawn. It is found that the mercury at the top falls to some point a and there rests; the height of a above b, the level of the surface of the mercury in the cup, is called the height of the barometric column, and measures the pressure of the atmosphere. At the level of the surface b inside and outside the tube the mercury would be in equilibrium of itself, and therefore the pressures must be equal from the column $a\,b$ inside the tube and from the atmosphere outside. The space from A to a is called the Torricellian vacuum because Torricelli first maintained that the height of the column $a\,b$ was the measure of the atmospheric pressure. Pascal demonstrated that to be the true explanation by taking the instrument up the mountain Puy de Dome, and observed that the surface a fell as the ascent was made up the mountain, and the portion of the atmosphere above the instrument became less. Thus one experiment settled the long controverted point of the cause of suction and the phenomena attributed to nature's abhorrence of a vacuum. The height $a\,b$ is found to be continually changing with the changing state of the atmosphere; it increases, or the barometer rises, when a cold dry north-easterly or easterly wind succeeds a warm moist south-westerly or westerly wind, and the converse. This is the direct consequence of the lower and more dense parts of the atmosphere changing in density, which being connected with

Fig. 37.

changes of the weather, the barometer is often called *the weather-glass*. As stated above, the barometer measures the pressure of the atmosphere, which is the weight of the vertical column of air at the place of the instrument, at that time. The average height of the barometric column in this latitude at the level of the sea is 29·98 inches, or 30 inches very nearly. There are small diurnal oscillations of the height, connected with the heat derived from the sun, which causes disturbance of the density, and motions in the atmosphere.

The barometer being applied to three different uses; first, as a weather-glass; secondly, as a meteorological instrument; and, thirdly, as a means of determining the heights of mountains with great accuracy; it is constructed accordingly. The weather-glass is generally an ornamental instrument with slight pretensions to accuracy, made with a bent tube like $A\,a\,b\,B$, fig. 38, closed at A and open at B. The surfaces of the mercury being a and b they move in opposite directions, with a change of the vertical height between them. A float of glass resting upon the surface b in the tube, being suspended by a thread attached to a wheel with a counterpoise to it, the wheel is turned round as the float rises or falls, and by means of an index on the face of the instrument tells the state of the barometric pressure, and the changes which indicate generally changes of the weather.

Fig. 38.
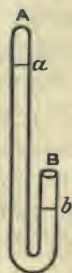
The barometers used for meteorological observations should be read off with certainty to one-hundredth of an inch at least, which is accomplished by a vernier applied to the scale at the upper part of the tube.

The mountain-barometer requires the best workmanship, for under a magnifying eye-glass, by the vernier and estimation, it should read to one-thousandth of an inch, and be so constructed as to be portable without derangement. To obtain the requisite accuracy, in all the better barometers now made there is a method of allowance for the change of the height of the

mercury in the cistern as well as in the tube. The best methods have the bottom of the cistern movable with a screw, and the level of the mercury within it is thus brought to the fixed standard height before an observation is made.

There have been other instruments invented for measuring the pressure of the atmosphere which have had their chief advantages in portability; such as the sympiesometer of Adie, the aneroid barometer of Vidi, and the excellent manometer of Bourdon. For the requisite tables to be used with a barometer, see a little treatise upon it by Mr Belville, of the Royal Observatory, Greenwich.

The atmosphere consists of about four-fifths nitrogen gas, one-fifth oxygen gas in volumes, with about one-thousandth part its volume of carbonic acid gas, and aqueous vapour in very various but small proportions. Sulphurous acid gas in places where pit-coal is largely used, and considerable quantities of carbonic acid gas in crowded rooms, are amongst the causes of local variations. From these considerations the air at any place not being of uniform composition its density is not given accurately from the pressure where great nicety is involved. The density is shown strictly only by the buoyancy produced by it, as in the experiment of fig. 36. Dr Prout found sensible changes of density, from unknown causes, affecting the atmosphere in 1832, which, as he suggested, might be connected with the cause of Asiatic cholera, then very virulent. The Aurora borealis is most probably caused by vaporous matter like the vaporous comets, and of like composition to the meteoric stones, which, coming into contact with the higher regions of the atmosphere in its motion through the planetary spaces, is made luminous by the earth's electro-magnetism, and, taking magnetic forms, becomes mixed with the atmosphere, and may slightly affect its density.

In simple gases the nucleus of each atom being surrounded by its atmosphere of caloric, electricity, &c. in equilibrium they must take a symmetrical or cubical arrangement. Since the cube which must be attributed to each atom is of different magnitude in different gases, therefore on being mixed the

equilibrium cannot be complete until they are uniformly diffused through each other. This takes place even when a light gas is placed over a heavier one, differing from the property of liquids discussed in Prop. 14, and also with respect to mixtures of gases and vapours.

Dalton expressed this property by saying that each gas acted as a vacuum to the others, so that there is a tendency to make a uniform mixture; but the complete mingling of the constituents is only accomplished after some time. From the continual disturbances from acting causes the atmosphere cannot be considered an absolutely homogeneous mixture.

PROP. 27. *To show that the density of the air decreases in a geometric progression for a series of heights in arithmetic progression.*

If we take a vertical column of the atmosphere above any place as A, fig. 39, and suppose it separated from the rest of the atmosphere by an imaginary rigid film, and take the area of the perpendicular section of the column equal to unity, we see that the pressure at any points, as A, P, or B, is the weight of the column above those points, and becomes less as we ascend. At first we suppose the temperature of the column the same everywhere.

Fig. 39.
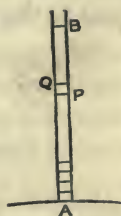
Let any height $AB = z$, and let this be divided into a very great number m of parts, and each equal to δ, or $\delta = \dfrac{z}{m}$.

If we take two neighbouring perpendicular sections P and Q distant δ, and let the height

$$AP = n\delta, \quad AQ = (n+1) \cdot \delta,$$

and the densities at these sections ρ_n and ρ_{n+1} respectively, and p_n, p_{n+1} the corresponding pressures: then the difference of the pressures $p_n - p_{n+1}$ is the weight of the elementary volume between the sections, and on account of the smallness of δ we

might take its density ρ_n or ρ_{n+1}. Since the temperature is considered constant, Boyle's law applies, and we have   *by subtraction*

$$p_n - p_{n+1} = g\rho_n\delta = \kappa\,(\rho_n - \rho_{n+1}),$$

or dividing by ρ_n, $\dfrac{\rho_{n+1}}{\rho_n} = 1 - \dfrac{g\delta}{\kappa}$ a constant ratio.

Putting $\beta = 1 - \dfrac{g\delta}{\kappa}$, if ρ is the density at the lowest station, we have the densities at the succession of heights δ, 2δ, 3δ, &c. equal to ρ, $\beta\rho$, $\beta^2\rho$, $\beta^3\rho$, &c. forming a geometric series with the common ratio β.

PROP. 28. *To find an expression for the difference of the pressures at* A *and* B; *and the difference of heights of two places, or the height* AB, *when the difference of pressures is known.*

Let p be the pressure at A, fig. 39, p' that at B, then the difference $p - p'$ is the weight of the column between A and B, or equal to the sum of the weights of all the elements;

or $\qquad p - p' = g\rho\delta\,(1 + \beta + \beta^2 + \beta^3 + \&c. \ldots + \beta^{m-1})$

$$= g\rho\delta\left(\frac{\beta^m - 1}{\beta - 1}\right) \text{ by summation,}$$

and substituting for β its value $1 - \dfrac{g\delta}{\kappa}$, we have

$$p - p' = g\rho\delta\,\frac{(1 - \beta^m)}{\dfrac{g\delta}{\kappa}}$$

$$= \kappa\rho\,(1 - \beta^m)$$

$$= p\,(1 - \beta^m),$$

or $\qquad \dfrac{p'}{p} = \beta^m = \left(1 - \dfrac{g\delta}{\kappa}\right)^m.$

Taking the logarithms on each side and substituting the expansion of $\log_\epsilon\left(1 - \dfrac{g\delta}{\kappa}\right)$ and $m\delta = z$, we have

$$\log_e\left(\frac{p'}{p}\right) = m\log_e\left(1 - \frac{g\delta}{\kappa}\right)$$

$$= -m\left\{\frac{g\delta}{\kappa} + \frac{g^2\delta^2}{2\kappa^2} + \frac{g^3\delta^3}{3\kappa^3} + \&c....\right\}$$

$$= -z\frac{g}{\kappa}\left\{1 + \frac{g\delta}{2\kappa} + \frac{g^2\delta^2}{3\kappa^2} + \&c....\right\};$$

and when δ is indefinitely small,

$$z = \frac{\kappa}{g}\log_e\left(\frac{p}{p'}\right).$$

Let h and h' be the heights of the barometer at A and B respectively, then $\dfrac{h}{h'} = \dfrac{p}{p'}$,

and $\quad z = \dfrac{\kappa}{g}\log_e\left(\dfrac{h}{h'}\right).$

This formula requires several corrections. First the temperature of the atmosphere diminishes 1° for every 100 yards of altitude in the lower atmosphere; secondly, the heights of the mercury in the barometer at the upper and lower stations, or h' and h, require correction for the difference of temperatures; thirdly, a correction is required for the moisture in the air; and, fourthly, the force of gravity g is slightly different in different latitudes.

As to the height to which the atmosphere reaches there have been widely different conclusions. From the duration of twilight it has been concluded that the height of the atmosphere in these latitudes does not exceed 45 miles; and this is the generally received height.

In the lower atmosphere the barometer is found to fall about one-tenth of an inch for every 30 yards increase of elevation, which may give an idea of the accuracy with which the heights of mountains may be determined by properly constructed barometers.

PROP. 29. *To find the height the earth's atmosphere would reach if everywhere of the same density as at the earth's surface.*

The pressure of the atmosphere being the same on a square inch as that produced by a column of mercury of 30 -inches height, if p be this pressure, ρ_m the density of mercury, ρ_a the density of the air at the earth's surface, H the height of the atmosphere supposed homogeneous,

we have $\qquad p = g\rho_m \times 30 \text{ inches} = g\rho_a H;$

$$\therefore\ H = \frac{\rho_m}{\rho_a} \times 30 \text{ inches.}$$

Taking the specific gravity of mercury $= 13\cdot6$, and that of air $= \cdot0013$, we have

$$H = 2\cdot5\,\frac{13\cdot6}{\cdot0013} = 10461\cdot5 \times 2\cdot5 \text{ feet}$$

$$= 8718 \text{ yards,}$$

$$= 5 \text{ miles nearly.}$$

PROP. 30. *To find the pressure of the atmosphere on a square inch at the earth's surface.*

The pressure of the atmosphere being measured by the barometric column, the pressure on each square inch equals the weight of a cylinder of mercury whose base is one square inch, and whose height is the height of the barometer, and it therefore is different at different times.

Taking the mean height of the barometer to be 30 inches, and the specific gravity of mercury $= 13\cdot6$, the pressure required equals the weight of 30 cubic inches of mercury, and equals $13\cdot6$ multiplied by 30 cubic inches of water,

$$= 13\cdot6 \times 30 \times \frac{1000}{1728} \text{ ounces,}$$

$$= 236\cdot1$$

$$= 14\cdot7 \text{ pounds,}$$

or nearly 15 pounds.

This result is frequently required to be employed where pressures are referred to that of the atmosphere.

The Magdeburgh hemispheres are two hemispheres to which handles can be screwed, one being furnished with a pipe and

stopcock, and the two fitting air-tight, they are exhausted by the air-pump, and the stopcock being then turned and the handles applied, it is found that great force is required to separate them.

Ex. Let the area of the circular section of the hemispheres be 10 square inches, or the diameter 3·56 inches, nearly, the force at each handle must be more than 147 pounds in order to separate them.

PROP. 31. *To find the height of the barometric column when the liquid employed is water; or to find the height of a water-barometer.*

Since the pressure of the atmosphere equals that produced by the column of water on a unit of area, let h' be the height of the water-barometer, and the density of water $= \rho'$, when the height of the mercurial barometer is h, and the density of mercury $= \rho$, also the pressure of the atmosphere on a unit of area $= p$,

$$\text{then} \quad p = g\rho h = g\rho' h',$$

$$\text{and} \quad h' = h \cdot \frac{\rho}{\rho'}.$$

If $h = 30$ inches, and $\frac{\rho}{\rho'} = 13\cdot6$,

$$\text{then} \quad h' = 2\cdot5 \times 13\cdot6 \text{ feet} = 27\cdot2 + 6\cdot8$$

$$= 34 \text{ feet.}$$

This result will be required in discussing the theory of *the siphon, the pumps,* and other instruments.

PROP. 32. *To find the height of an oil-barometer.*

This question is to be worked as the last, and if the specific gravity of the oil were ·94, then

$$\text{the height of the oil barometer} = 2\cdot5 \times \frac{13\cdot6}{\cdot94} \text{ feet}$$

$$= 36\cdot2 \text{ feet nearly.}$$

The results of questions analogous to the last three propositions will be found in examples 7, 8, 9, to Chapter I. Baro-

meters have been made of water as well as oil, for the purpose of having enlarged scales of the changes of height of the columns, but they have the disadvantage of requiring an allowance to be made for the elastic force of their vapours in the Torricellian vacuum. The elastic force of the vapour of mercury in that of the ordinary barometer is inappreciable, although there is no doubt of its existence.

On Pneumatic Instruments.

There have been numerous different constructions of air-pumps invented, but two will perhaps only remain in general use in this country; namely, the common table air-pump for ordinary experiments, and Newman's simple and effective air-pump where a high degree of exhaustion is required.

The common table air-pumps have their valves made in a very simple manner, by tying a strip of oiled silk or thin bladder, half an inch broad, over an aperture such as A in figure 40, of the small pipe AB passing through a plug of brass which screws into the bottom of the barrel of the pump for the lower valve and into the piston for the higher valve of the pump. The pressure under the oiled silk will

Fig. 40.

cause it to rise and allow air to escape through AB from below, but falling upon the aperture it prevents the air returning.

Its lightness is of advantage in procuring a considerable degree of exhaustion, and it is strong enough to bear the pressure of the atmosphere on the small area of the pipe; but it requires to be occasionally renewed to keep the instrument in effective condition.

PROP. 33. *To explain the construction of the common double-acting air-pump.*

Let AB and CD in fig. 41 represent the cylindrical barrels of the pumps open to the air at B and D, and connected at A

and C with the pipe EFG leading to G the middle of the plate

Fig. 41.
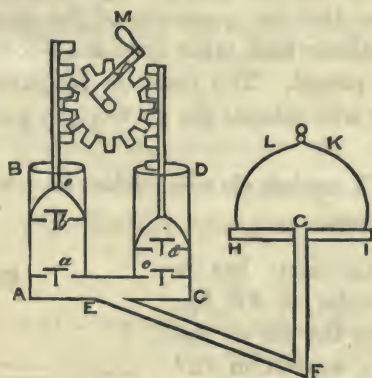
of the air-pump HI, on which the glass receiver as KL, or other apparatus, is placed, which it is required to exhaust of air.

Let a, b, c, d represent the valves opening upwards, of which b and d are in the air-tight pistons which are raised and lowered by the toothed rods on which the toothed wheel of the figure acts, as it is turned round backwards and forwards by means of the handle M.

Let the piston $b\,e$ be at the bottom of the barrel, and then being raised the valve b closes the aperture and prevents the external air from entering, whilst the air in the barrel being rarefied, the elastic force of the air under the valve a raises it up and the air rushes from the receiver and pipes to fill the barrel AB. When the piston descends again the valve a closes and prevents the air in the barrel from returning to the receiver, and the air in the barrel being compressed by the descent of the piston it raises the valve b and escapes outwards. The same takes place in the barrel CD as the piston is raised and lowered, and in this manner a portion of air is withdrawn from the receiver at each stroke of the pistons, as long as there is elastic force sufficient in the remaining air to raise the valves. When the elastic force becomes insufficient to raise the valves the pumps cease to act, and no further exhaustion can be obtained.

The lightness of the valves and accurate fitting of the pistons
are evidently points of consequence in the construction; and
the labour of working the pumps is reduced by having two
pumps acting, so that the pressures of the atmosphere on the
pistons counterbalance each other upon the wheel and the winch
by which it is turned. The common air-pumps are in good
order when they will exhaust the air to $\frac{1}{120}$th part.

PROP. 34. *To explain the construction and mode of action of
Newman's air-pump.*

Newman's air-pump has a
single large cylinder as *AB*, fig.
42; it is open to the air at the
top *B*, but has a separation *CD*
through which the piston-rod
works, and with a metal valve *b*
in its opening upwards; and above
the separation is a quantity of oil.
The piston is solid as *EF*, and
has a metal valve *a* in it also
opening upwards. The pipe *G*
leading to the plate of the air-
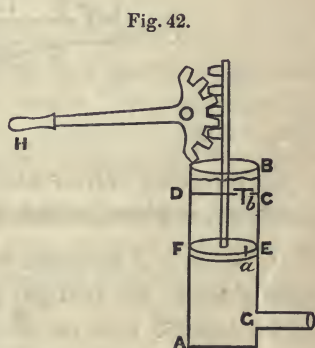
Fig. 42.

pump is at such a distance from the bottom of the cylinder
that the piston *EF* passes below it at each stroke. The piston
being raised and lowered by means of the lever with handle
H, and toothed arc acting upon the toothed rack of the piston-
rod, when brought up to the separation *CD* the air above *EF*
is forced through the valve *b* and the oil above it, and when
moved down again the air below may raise the valve *a* and pass
into the barrel, or when it becomes very rare may only fill the
barrel when *EF* passes below the pipe *G*; but the pump will
continue to act as long as any air can be lifted through the valve
b and the oil above it. When in good order this pump will
exhaust the air to less than $\frac{1}{1000}$th part of the original air in the
receiver.

PROP. 35. *To find the quantity of air in the receiver after
a given number of strokes of the piston of an air-pump.*

Supposing the valves to act perfectly, let a be the volume of the air in the receiver and pipes at commencing, b the volume of the barrel. Let ρ be the original density of the air,

ρ_1 the density after the first stroke of the piston,

ρ_2 .... second ....

&c. .... &c. ....

Then since the volume a is expanded to the volume $a + b$ by raising the piston,

$$\rho_1 (a + b) = \rho . a, \quad \text{or} \quad \rho_1 = \frac{\rho}{1 + \dfrac{b}{a}}.$$

Similarly

$$\rho_2 (a + b) = \rho_1 a, \quad \text{or} \quad \rho_2 = \frac{\rho}{\left(1 + \dfrac{b}{a}\right)^2};$$

and so onwards, or after the nth stroke we have

$$\rho_n = \frac{\rho}{\left(1 + \dfrac{b}{a}\right)^n};$$

and we see that the density of the air in the receiver decreases in a geometric progression, but never becomes zero even if the valves were perfect.

Prop. 36. *To explain the construction of the siphon and barometer-gauges of the air-pump.*

The air-pump gauges are instruments for showing the degree of exhaustion which has been attained, by exhibiting the elastic force of the air remaining in the receiver through the height of the mercurial column which it will support.

For the common air-pump the siphon-gauge is generally used, and it is made of a bent tube of glass like fig. 43, closed at one end as A and open at the other B. It is filled with mercury from A to some point a, and being screwed air-tight in a vertical position to the pipes of the air-pump, with

Fig. 43.
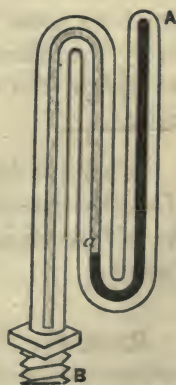
which its open end B is in communication, then, when the elastic force of the remaining air in the receiver will not support the height of the column of mercury between the levels of A and a, say about 2 inches, the mercury falls from the end A, and as the exhaustion proceeds the levels of A and a come nearer and nearer together. If the level of the surface in the closed leg is only $\frac{1}{4}$th inch above that in the other, the elastic force of the remaining air is only $\frac{1}{120}$th that of the original quantity if the barometer stands at 30 inches.

The siphon-gauge should not be depended upon in accurate experiments, because by usage small quantities of air will pass into the closed leg and may be often seen forming a small bubble at A by means of a magnifying eye-glass.

For accurate experiments the barometer-gauge should be always employed. It consists of a straight tube of glass 31 inches or more in length, the upper end being cemented into a brass cap which communicates with the pipes of the air-pump by its open end C, fig. 44. The lower end dips into a cup of mercury as B in the figure. When the air-pump is used the internal air of the receiver and pipes being rarefied, the mercury rises from the cup into the tube, say to some height A. If a perfect vacuum could be obtained in the receiver the height AB would be the same as in the barometer, and when any degree of exhaustion is produced we know the elastic force of the remaining air by the difference between the height AB and the height the mercury stands in a good barometer.

Fig. 44.
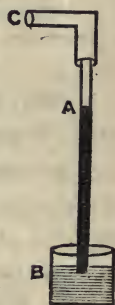
When the difference between the height AB and the height of the barometer, standing at 30 inches, is $\frac{1}{50}$th of an inch, then the air remaining in the receiver is $\frac{1}{1500}$th of the original quantity.

PROP. 37. *To explain the construction of the condensing syringe.*

The condensing syringe consists of a cylin-
drical barrel as *AB*, fig. 45, in which an air-tight
piston works by a rod from the handle *C*. The
valves *a* and *b*, the former at the further end of
the barrel and the latter in the piston, both open
outwards, so that as the piston is forced down, the
valve *b* closing, the contained air is forced through
the valve *a* into a receiver screwed to the end *D*,
and is prevented returning by the valve *a* closing
as the piston is drawn up again, and a fresh supply
of air enters the barrel through the valve *b*, which
opens to admit it.

Fig. 45.
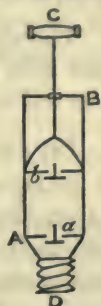
If the syringe acts perfectly and the same quantity is forced
into the receiver at each stroke of the piston, then the quantity
of air in the receiver will evidently be in an arithmetic pro-
gression as the syringe is worked.

If the receiver be furnished with a stop-cock, by weighing it
when filled with condensed air, and measuring the air which
escapes on the stop-cock being opened, and then weighing the re-
ceiver again, we obtain the weight of the air which had escaped.

The weight of a given volume of gas is also obtained by
having a spherical receiver filled with the gas of which the weight
is known, then exhausting with the air-pump and weighing
again. In experiments to determine the weight of hydrogen gas
many precautions are necessary on account of its great lightness.

The condenser or pump, and the receiver, of the air-gun are
now made differently to the above. The pump has a solid
metal plug-piston, which requires only oil to keep it air-tight in
the barrel, and neither the barrel nor piston have any valve in
them, the fresh supply of air entering the barrel through a hole
in its side near the end when the piston is drawn back. The
receiver has a strong metal valve to prevent the air which is
forced into it from escaping. When the air-gun is used the
charged receiver is screwed to it, and a strong bent steel spring
being let off by the trigger it forces a steel pin against the valve
of the receiver, so as to open it and allow a part of the con-
densed air to escape, and so to drive the charge from the barrel.

PROP. 38. *To explain the construction and mode of action of the siphon.*

The siphon however varied in form consists essentially of a bent tube of glass or metal, and its use is to remove a liquid from one vessel to another.

Let aCb, fig. 46, be the bent tube or siphon, AB the surface of the liquid in the vessel. Then the siphon being in the first place filled with the liquid, and the ends a and b kept closed, until it is put into the position of the figure, if we draw a horizontal line ABd and the height of the highest point C above it is less than the height of a barometer formed of that liquid, then the fluid will not break at C, but the portions on each side of the highest point to the horizontal line ABd will balance each other. The liquid in the tube from d to b will be however unbalanced, and will by its weight flow out of the tube, and since there will be no break at the highest point, the pressure of the atmosphere, which is nearly equal at the surface AB and the orifice b, will cause an equal quantity to enter the tube to that which flows out, and thus the tube being always kept full the flow will continue until the surface AB descends below the level of either a or b, in either of which cases the flow will cease.

Fig. 46.
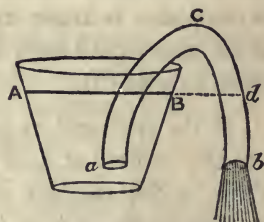
It is clear that the limits of the height on the average of a siphon for mercury cannot exceed 30 inches, nor of one for water, 34 feet; but for alcoholic spirit might be higher according to its specific gravity. No siphon of course can act in a vacuum, the pressure of the air or other elastic fluid being necessary in order to keep the tube filled with liquid.

The experiment called Tantalus' cup is a vessel containing a siphon, of which one end opens into the vessel near the bottom, and the other end passing through the bottom of the vessel opens below. When filled above the bend of the siphon the liquid flows out again and the vessel empties by the action of

the siphon. Tides-wells, or reciprocating springs, are caused by natural siphons in the rocks connecting the well and its source of supply; the flow ceasing when the siphon has drained the source of supply and recommencing when the water rises again above the bend of the siphon.

In the different kinds of pumps for raising water or other liquids, the pressure of the atmosphere acts to keep the barrel filled with liquid from the well or reservoir as they are ordinarily constructed. The suction-pump is one which depends chiefly for its mode of action on the pressure of the atmosphere. The lifting pump is one which acts chiefly by lifting a column of water. And the forcing pump acts chiefly by forcing upwards a quantity of water.

PROP. 39. *To explain the construction and mode of action of the suction-pump.*

In figure 47 let AB represent the barrel of the suction-pump, C the opening of its pipe below the sur-face of the water in the well or cistern, D the spout or exit-pipe, and EF the lever by which the piston or bucket of the pump is raised and lowered. Let a be the valve at the bottom of the barrel opening upwards, and b a like valve in the piston. On the piston being worked up and down, it will if air-tight first ex-haust the air from the barrel and pipe leading to the cistern, in like manner to the air-pump, and the pressure of the atmosphere upon the surface of the water in the cistern will cause it to rise up the pipe into the barrel, provided it be not more than about 34 feet, on the average, above the water in the cistern. When the water enters the barrel its return is prevented by the valve a falling and closing the opening, and as the piston descends into the

Fig. 47.
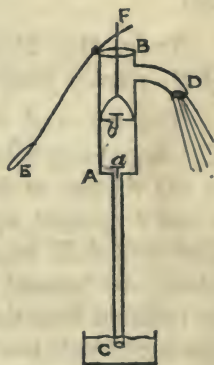
water the valve b opens to allow it to pass above the piston; then on the ascent of the piston the water is lifted in the barrel and finally flows out by the pipe D; a fresh supply filling the barrel from the pressure of the atmosphere upon the surface in the cistern.

PROP. 40. *To explain the construction and mode of action of the lifting pump.*

Let AB be the level of the surface of the water in the well or cistern, CD the cylindrical barrel of the pump, having a pipe descending below the surface AB, as in fig. 48. Let a be the valve at the bottom of the barrel, b that in the piston, and c another in the exit-pipe, all opening upwards. The top of the barrel being closed, the piston-rod EF works through a water-tight stuffing box d. The pump with its valves and piston acts as in the cases before described; the water passing up the exit-pipe GH is prevented from returning by the valve c falling and closing the aperture, and the water may thus be lifted to any height when a sufficient force is applied to the piston-rod EF.

Fig. 48.
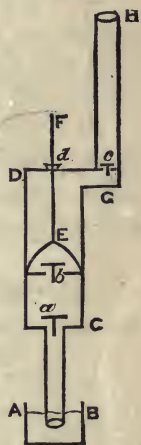
PROP. 41. *To explain the construction and mode of action of the forcing pump.*

The barrel of the forcing pump is open at the top, with a pipe descending below the surface AB of the water in the cistern, as in fig. 49. It has two valves a and b opening upwards, and an exit-pipe CD. The piston E is solid, without a valve, being raised and lowered by the piston-rod EF. The air is forced from the barrel on the descent of the piston through the valve b, and then the barrel being exhausted on the ascent of the piston, the pressure of the atmosphere on the surface AB will cause the water to rise in the pipe and barrel to some height, not exceeding about 34 feet; and being prevented returning from the barrel by the

Fig. 49.
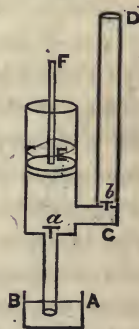
valve a falling and closing the aperture, then on the descent of the piston again it is forced through the exit-pipe CD, and prevented returning by the valve b closing the aperture.

By this pump water may be forced to any elevation when sufficient force can be applied.

PROP. 42. *To explain the construction and mode of action of the common fire-engine.*

The fire-engine consists of two forcing pumps and an air-vessel to maintain a continuous jet of water from the nozzle of the exit-pipe.

Let AB be the surface in fig. 50 of the water in the cistern, from which the pumps are supplied; CD and EF the barrels of

. Fig. 50.
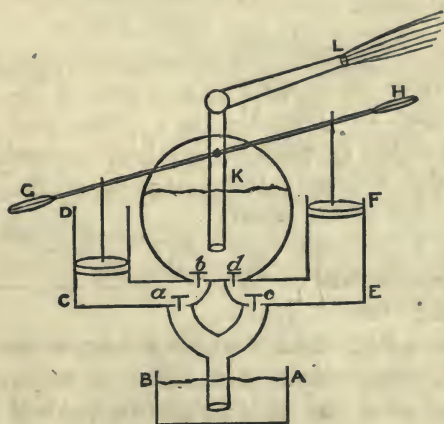
the forcing pumps, the pistons of which are worked by the lever GH. Let a, b, c, d be the valves of the forcing pumps, of which b and d open into the air-vessel K. Then the exit-pipe coming to near the bottom of the air-vessel, when the pumps are working the contained air becomes compressed, and the water as in the figure occupies a considerable part of the vessel, and thus the elastic force of the compressed air acting upon the water produces the air-pump, which maintains a continuous jet of water from the nozzle L of the pipe.

If the pumps communicate by a rigid pipe with the source of
water supply they will draw up by the pressure of the atmo-
sphere, as other pumps do, the water they require; but if con-
nected only by *a flexible* pipe there must be a pressure of the
water supplied to keep the pipe distended, and to cause the
water to flow along it, from either another engine or from a
higher level or head of water.

There have been many forms of machines invented for
raising water and called pumps, which are more properly dis-
cussed in treatises on practical mechanics than in hydrostatics,
such as the centrifugal pump and the chain-pumps, but Ar-
chimedes' screw may properly find a place amongst the ordinary
pumps.

PROP. 43. *To explain the construction and mode of action
of Archimedes' screw.*

Archimedes' screw consists of
a pipe wrapped in a spiral
round a cylinder, as in fig. 51,
and the axis of the cylinder is
inclined to the horizon. Let
AB be the surface of the water
in the cistern, a the lower open
end of the pipe, which on the
cylinder being turned round
passes into the water, which then fills the lower part of the pipe.

Fig. 51.

On the end a being turned to the place b, the water will have
moved to the lowest part of the cylinder, but will have reached
a higher part of the pipe, and on the cylinder being continuously
turned in the right direction, the water will ascend and finally
flow from the upper open end c. In this manner Archimedes'
screw raises water from a lower to a higher level without the
friction of the pistons of other pumps and without valves, but
the quantity raised is not large compared with the size of the
machine. There is evidently a limit to the inclination of the
cylinder to the horizon compared with the inclination of the
pipe to the axis of the cylinder, in order that the parts of
the pipe at the lower side of the cylinder in any part of a revo-

lution may be lower than those on each side of them on the upper side of it.
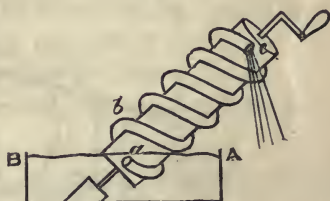
PROP. 44.   *To explain the principle of the pneumatic trough and diving-bell.*

Let *ABCD* be a vessel in fig. 52, containing a liquid, of which the surface is *AB*. If a vessel as *EF* or *GH* open at one end and closed at the other be filled with the liquid, and the closed end be turned upwards, it may be brought above the surface, and the vessel will remain filled with the liquid, provided its length is not more than the barometric column for that liquid, arising from the pressure of the atmosphere.   A vessel of glass, for instance, as *GH*, may be placed with its open end *G* upon a shelf of the trough *GK* below the surface *AB*, and remain full of the liquid. If air or any gas be now forced under the open end at *G* it will rise through the liquid, and occupy the upper part of the vessel, as for instance from *H* to *ab*.   If the vessel is graduated, the volume of the gas at the atmospheric pressure is known by bringing the surfaces of the liquid inside and outside the vessel *GH* to the same level, and reading off the graduation at the level.   If the surface inside as *ab* is higher than *AB*, the contained gas is subjected to less than atmospheric pressure, and if at *cd* in the vessel *EF* it is below *AB*, it is subjected to greater than atmospheric pressure.   The volumes *Hab* and *Fcd* can be calculated by Boyle's law, when the volume at the atmospheric pressure and the heights of *ab* and *cd* above and below *AB* are given, together with the density of the liquid.
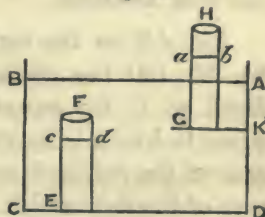
If the vessel *EF* or *GH* filled with air or gas were immersed directly in the liquid with the closed end upwards, we should find the volume, the air or gas, occupied by Boyle's law when the depth of immersion was known, and *EF* would then represent the case of the diving-bell.

It is easy to see that gases may be kept in vessels within troughs of liquids which do not absorb them or on which they

do not act, and may be measured with accuracy, mixed in given proportions, and the results of such mixtures examined.

PROP. 45.   *When a diving-bell is of a prismatic or cylindrical form, to find the part of the bell which will be free from water when sunk to a given depth, and no fresh supply of air has been admitted.*

Let AB be the surface of the water in fig. 53, CD the diving-bell, and AC the depth of C, the top of the bell below AB equal to h feet. Let CD the height of the axis of the bell in feet $= a$, and $CM = x$ the part of the axis which is vertical, above the water in the bell at M. If V is the whole volume of the bell, V' the volume the air occupies at the depth $AM = h + x$, then if we take the atmospheric pressure equal to that from a column of 34 feet of water, we have by Boyle's law, whatever be the form of the bell,

Fig. 53.

$$\frac{V'}{V} = \frac{34}{34 + h + x}.$$

When the bell is cylindrical or prismatic $\dfrac{V'}{V} = \dfrac{x}{a}$, and we have the quadratic equation

$$x\,(34 + h + x) = 34a$$

to determine x in feet.

PROP. 46.   *If two gases which do not act chemically upon each other are mixed in a vessel in the pneumatic trough, to show that the product of the elastic force multiplied into the whole volume of the mixture equals the sum of the products of the elastic forces into the volumes of the components.*

The gases having formed a complete mixture, let ρ, e, V be the density, elastic force, and volume of the mixture; ρ', e', V' those for one of the components, ρ'', e'', V'' those for the other. Then since tne mass is constant, we have

$$\rho V = \rho' V' + \rho'' V'';$$
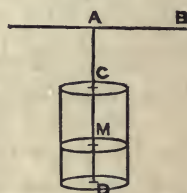
and by Boyle's law the elastic force is proportional to the density;

$$\therefore \; eV = e'V' + e''V''.$$

Cor. The same may be continued to any number of gases, or

$$eV = \Sigma \, (e'V').$$

Examples in Pneumatics.

Ex. 1. A cylindrical tube closed at one end, and 20 inches long, with its perpendicular section one square inch, has an air-tight piston at the open end. When the barometer stands at 30 inches and the specific gravity of mercury is 13·6, shew that a force of 26·2 ounces must be applied at the piston to force it down 2 inches into the tube, and that a force of $22\frac{1}{8}$ pounds must be applied to force it to 8 inches from the closed end.

By Boyle's law we have $\dfrac{e'}{e} = \dfrac{V}{V'}$, therefore $e' - e = e\left(\dfrac{V}{V'} - 1\right)$ = the pressure to be applied to change the volume from V to V'. Applying the result of Prop. 30 to the first case, we have

$$e' - e = 236{\cdot}1 \left(\frac{20}{18} - 1\right) \text{ ounces} = 26{\cdot}23 \text{ ounces.}$$

In the second case

$$e' - e = 236{\cdot}1 \left(\frac{20}{8} - 1\right) \text{ ounces} = 22{\cdot}13 \text{ pounds.}$$

Ex. 2. If the tube in the last question had been 30 inches long, shew that the results would have been respectively 16·86 ounces and 40·57 pounds.

Ex. 3. If an air-bubble, which is a sphere of $\dfrac{1}{100}$th inch diameter at a depth of 102 feet in water, ascends to a depth of 68 feet, show that its diameter is then $\dfrac{1}{90{\cdot}8}$th inch, and when it ascends to a depth of 34 feet its diameter is $\dfrac{1}{79{\cdot}4}$th inch.

Ex. 4. If the volume of a gas is 100 cubic inches at the temperature $60°$ Fahrenheit, what will be its volume under the same pressure at $16°$ below the zero of Fahrenheit's scale?

If V_0 be the volume at the freezing point,

V .... $60°$ temperature,

V' .... $-16°$ ....

then

$$V = V_0 (1 + \alpha t^0) = V_0 \left(1 + \frac{28}{494}\right),$$

$$V' = V_0 (1 - \alpha t'^0) = V_0 \left(1 - \frac{32 + 16}{494}\right);$$

and since by dividing $\dfrac{V'}{V} = \dfrac{1 - \alpha t'^0}{1 + \alpha t^0}$,

$$\therefore \ V' = V \frac{223}{261} = 85\text{·}44 \text{ cubic inches,}$$

since $V = 100$ cubic inches.

Ex. 5. If the density of the atmospheric air is called unity at the freezing point, what is its density at the boiling point of water?

Since generally $\dfrac{\rho'}{\rho} = \dfrac{V}{V'}$, also 100 measures of air at the freezing point become $136\text{·}4$ at the boiling point; therefore if $\rho = 1$, then ρ' at the boiling point is

$$\rho' = \frac{100}{136\text{·}4} = \text{·}733.$$

Ex. 6. If the volume of a gas is 100 cubic inches at the freezing point of water when the barometer stands at 30 inches, what will be the volume when the barometer stands at 29 inches and the thermometer at $50°$?

By Amonton's law $\quad p = \kappa \rho (1 + \alpha t^0), \quad p' = \kappa \rho' (1 + \alpha t'^0);$

$$\therefore \ \frac{\rho}{\rho'} = \frac{V'}{V} = \frac{p}{p'} \cdot \frac{1 + \alpha t'^0}{1 + \alpha t^0}.$$

To find V', let $V = 100$ cubic inches, $\dfrac{p}{p'} = \dfrac{30}{29}$, $t^0 = 0$, $t'^0 = 18°$,

$\alpha = \dfrac{1}{494}$, then $V' = 107\text{·}21$ cubic inches.

Ex. 7. A tube which is cylindrical and 40 inches long, with one end closed, has mercury poured into it to fill 30 inches of its length, then the open end being covered with the thumb, it is inverted with the covered end below the surface of mercury in a cup, and the thumb is withdrawn; required the height the mercury stands in the tube, the height of the barometer being 30 inches.

Let x be the height the mercury stands in the tube in inches, then $40 - x$ is the height occupied by the air. The barometric column was 30 inches when the volume of the air occupied 10 inches of the tube, and when inverted the pressure is that from $30 - x$ inches of mercury upon the contained air. Therefore since $\dfrac{V'}{V} = \dfrac{p}{p'}$ by Boyle's law, we have

$$\frac{40 - x}{10} = \frac{30}{30 - x},$$

which gives $x = 16 \cdot 973$ inches.

Ex. 8. A cylindrical tube closed at one end has an air-tight piston at 14 inches from the end, and it requires a force of $13\frac{1}{8}$ pounds to draw the piston to 18 inches from the end when the pressure of the atmosphere is 15 pounds upon the square inch; show that the area of the surface of the piston is 4 square inches.

Ex. 9. If the receiver of an air-gun has a volume five times that of the barrel of the pump, show that if the pump acted perfectly it would require forty-five strokes of the piston to charge the receiver to ten atmospheres.

Ex. 10. Shew that in a mine upon a mountain where the barometer stands at 26 inches of mercury, the height of the lower valve of a pump cannot be more than $29\frac{7}{15}$ feet above the water in the mine.

CHAPTER IV.

ON HEAT.

IN the introductory chapter it was stated, that caloric, as the cause of heat, was an essential part of all bodies as we meet with them; and that changes of the state of dense matter accompany changes of the amount of caloric which it contains. Sometimes the change is from a solid to a liquid form, or from either to a vaporous form, and the converse; but generally, though not universally, increase of the caloric of a body is attended by an increase of its bulk, whether it is solid, liquid, or gaseous. The exceptions occur in liquids coming near their temperatures of solidification, as water and some metals, which expand again as they cool before they become solid. Crystals are found to expand unequally in different directions on being heated.

It was discovered by Dr. Black that heat disappeared on the change of matter from the solid to the liquid, or from the liquid to the vaporous state, and he called the heat which had disappeared, *latent heat*. Thus it requires a large quantity of heat to convert ice at the freezing temperature into water at the freezing temperature, and also to convert water at the boiling temperature into steam at the boiling temperature. It is in this respect that the protoxide of hydrogen becomes ice, water, or steam, according to the quantity of caloric it possesses, and which is essential to each state. What would be the state of dense matter without caloric we cannot tell; but, reasoning from analogy, we imagine that bodies would be exceedingly small compared with their bulks as we meet with them, and of a hardness compared with which that of diamond is softness. The theory of latent heat must be considered in conjunction with the known fact that it will leave the body on the temperature diminishing to a certain amount, as when water turns into ice at temperatures below the

freezing point, and has then given out its latent heat of liquidity, and in other like cases. In this way if we call the latent heat combined caloric, we must understand the combination of caloric and dense matter as essentially of a different nature to that of an acid and a base, which produce a salt with properties of its own differing from those of its components. We have no evidence to conclude that caloric forms definite compounds with dense matter, although it may do so, and give to it peculiar properties in cases which are at present beyond our means of research. That it is held by affinity for dense matter in bodies we cannot doubt, but from its atoms being highly repulsive of each other, it leaves such dense matter, unless restrained by the opposite repulsion of other caloric. We shall see further on that this repulsion must be considered in connexion with the radiation and the temperature of the body.

By the temperature or heat of a body we mean that degree of heat which affects the senses, and which is shown by the thermometer or pyrometer; the degree of heat being measured by the bulk, or relative bulks, of some bodies of which the instrument is made. In the common mercurial thermometer, the mercury in the bulb and stem expanding and contracting much more than the glass in which it is contained, from an increase or diminution of heat respectively, the change of temperature is shown by the portion of the tube in the stem which is filled with the mercury. By experiments we find easily that the equal changes of temperature produce equal or nearly equal changes in the height of the mercury in the thermometer, and hence we receive it as an instrument for measuring the temperatures within its range. For all ordinary purposes a well constructed mercurial thermometer gives correct indications of the temperatures of its scale, and its degrees show equal changes of temperature at different parts of the scale for such uses. The thermometers made with spirit of wine in glass, and those made with air in glass, however, show some small but sensible differences from the mercurial thermometer, and it becomes a question of some importance to find an accurate thermometric scale of degrees which shall indicate equal increase of temperature by the equal increase of degrees on the scale. The gases are found to ex-

pand more uniformly than liquids and solids; and when these latter are tested by the air-thermometer, they are found to expand more at the higher temperatures than at the lower ones for equal increase of temperature; and hence the mercurial and spirit-thermometers are not rigidly accurate instruments, if graduated upon the supposition of uniform expansion of the mercury or spirit.

If the expansion of the gases is to be tested, we need an absolutely correct scale for comparison with them, and this is not to be found. We have, however, the alternative of comparing them with each other, as, for instance, by forming two air-thermometers, one with a gas which has never been liquefied by cold and pressure, and the other with one which is easily reducible to the liquid state. If the latter showed a divergence from uniform expansion, as compared with the former, we might take the result as applying to others in like circumstances.

Regnault, by weighing the volumes of the gases contained in known glass spheres, found the result given at the beginning of Chapter II. The formula of Gay Lussac's law for the volumes of the gases at different temperatures being as in Chapter II. $V = V_0(1 \pm \alpha t^0)$, where V_0 is the volume at the freezing point of water, and V the volume at t^0 above or below that point, the values of V will evidently form an arithmetic series for equal increments and decrements of temperature. We have $V = 0$ when $-t^0 = \dfrac{1}{\alpha}$, now taking $\alpha = \dfrac{1}{494}$ for atmospheric air, then when $t^0 = -462^0$, we have $V = 0$, or the gas has become annihilated in volume, and therefore in existence, whatever the pressure might be. We should call this the absolute zero of the thermometric scale; its distance, however, below the freezing point of water would be different for the different gases, since α is different for each of them; and we must conclude that Gay Lussac's law is only an approximation sufficient for use within moderate limits of temperature.

Dalton considered the volumes of the gases to form a geometric series for temperatures in an arithmetic series, and such a series will differ very slightly from an arithmetic one, when the common ratio differs only slightly from unity. Let V be the

volume at any temperature, δV the increase of volume for 1° increase of temperature; then, according to Dalton's view, $\dfrac{\delta V}{V} =$ constant. Putting this expression into a differential form and integrating, we find $V = V_0 \epsilon_t^{at^0}$, where a is the constant, and ϵ is the base of the Napierian logarithms. Expanding the exponential ϵ^{at^0}, we have

$$V = V_0 \left(1 + at^0 + \frac{(a^2 \cdot t^2)}{1 \cdot 2} + \frac{(a^3 \cdot t^3)}{1 \cdot 2 \cdot 3} + \&c.... \right).$$

Let $t^0 = 1^\circ$, and then

$$V = V_0 \left(1 + a + \frac{a^2}{1 \cdot 2} + \frac{a^3}{1 \cdot 2 \cdot 3} + \&c.... \right);$$

and for air $\dfrac{V - V_0}{V_0} = \dfrac{1}{494} = a$, since we may neglect the higher powers of a. For moderate ranges of the scale near the freezing point of water, the formula becomes

$$V = V_0 (1 + at^0) \text{ nearly,}$$

or agrees nearly with Gay Lussac's; but it leads to wide differences for extreme cases, and does not involve the absurdity of the air becoming of volume zero at 494 degrees below the freezing point of water; of which the connexion with nitrogen and other gases is inconceivable.

If we take the expression $V = V_0 \epsilon^{at^0}$ when $t^0 = 0$, we have $V = V_0$; and when t^0 is negative $V = V_0 \epsilon^{-at^0}$, which is less than V_0, but only becomes zero when t^0 is minus infinity. It appears that we may take any point for starting point; for if t^0 were the temperature when the volume is V', then $V' = V_0 \epsilon^{at'^0}$; let $t = t' + t''$, then $V = V_0 \epsilon^{at^0} = V_0 \epsilon^{at'^0} \cdot \epsilon^{at''^0} = V' \epsilon^{at''^0}$, of the same form as the original one; and thus we avoid the absurdity of the gas vanishing at a particular temperature depending upon the value of a for that gas and the freezing point of water.

The formula $V = V_0 \epsilon^{at^0}$ would give a greater degree of expansion at the higher temperatures, as seen in the series of the expansion; for the terms involving the higher powers of t^0 will then have sensible values.

The gases are evidently the vapours of solids or liquids which have great affinities for caloric, and far removed from their dew-points, or the temperatures where they would become liquids exhibited in dew.

It is probable that liquids may have their volumes expressed with a like law, showing greater expansion at higher temperatures; but if they crystallize, or their atoms take peculiar arrangements with regard to each other when they become solid, then there will be an additional term in the expression, as on approaching the solid state and losing their liquidity, their atoms commencing peculiar arrangements, the volume may *increase* for *diminishing* temperatures from the temperature of maximum density. Water is found to be at the greatest density at $7^\circ\cdot1$ above freezing, or at $39^\circ\cdot1$ Fahrenheit. A strict formula for liquids must evidently involve this consideration, and also a term for the value of their attraction of aggregation.

Though we may not know the *absolute* quantities of caloric in bodies, yet we have the means of finding the *relative* quantities which they give out and absorb in passing through given temperatures; these will be found discussed under '*specific heats;*' and it is presumed that the quantities of caloric which they contain are represented relatively by the same numbers which represent their specific heats. This supposes that the specific heats, or capacities for caloric, are constant at all temperatures, which there is great reason to doubt.

As the volume of a body depends, when free, upon the temperature and quantity of caloric which it contains, we have the reciprocal result of a change of these when we forcibly change the volume of it. It is a well-known fact that iron when briskly hammered becomes red hot; but when once hammered and condensed in bulk, it does not exhibit the same result on being hammered again, unless it has been brought to its original state by annealing in the fire. Other metals also show like results; the drawing of wire and the rolling of plates of metal produce heat. Other bodies generally become heated on being compressed, but iron affords the advantage of large specific heat, and strong tenacity to bear the blows of the hammer. The gases give out heat on being greatly compressed, and in the fire-

syringe it is sufficient to fire tinder. We here meet with considerations of the *defective* elasticity of bodies, which was discovered by Professor Eaton Hodgkinson to be a general property of solids; that is, they never recover *immediately* and *perfectly* their original form after a strain, however small; and he considers liquids and gases to be also subject to the same law, but in a much less degree. If the hammered iron, in the above-mentioned experiment, had recovered its original form by virtue of its elasticity, after each blow of the hammer, we can have no doubt it would never have become heated. The minute sparks of red-hot steel, struck by a flint from steel, occur in the same way; but experiments have been tried where the condensation was only temporary, yet heat was given out to the condensing body, without sensible change of structure in either body. Thus a smooth metal disc rotating, with a piece of smooth metal pressing upon it, the latter becomes heated without any sensible abrasion from either surface. In this case the pressure produces a slight compression of the disc, and the rubbing body becomes heated by each successive part of the disc, which recovers its lost heat during the remainder of the revolution. There has been no case brought forward where condensation may not have produced the heat which is witnessed, like that of the hammered iron, which is the normal experiment.

We conclude that the temperature of a body, its capacity for caloric, and the amount of caloric in it, are connected together. If the amount of caloric remains the same, the temperature of a body is increased when its capacity for caloric is diminished, and the converse.

The caloric may pass from one body to another in two different ways; first, by *radiation*, that is, by rays of heat (such as the rays of light from a shining body) coming from one body to another; secondly, by *conduction*, when the heat passes from one body to another in contact with it, or from one particle to a neighbouring one of the same body.

The laws of radiant heat are found to be the same as those of light; that is, it is reflected, refracted and polarized, according to the same laws. These are easily shown with regard to the heat accompanying the light of the sun, but require more care in

their demonstration for the heat of a common fire, and of non-luminous bodies.

With regard to the conduction of heat, it is very different for different bodies; and the metals have been shown to conduct heat in the same order in which they conduct electricity.

That the radiant light and heat from the sun are reflected and refracted according to the same laws, is shown by the focus of a concave mirror held in the sun's light being the same for the heat as for the light, or they are reflected alike; and when a convex lens is held in the sun's light, the focus of the heat is at the same point or very near it, as that of the light, or they are refracted by the glass in a like manner. That the radiant heat of the sun may be polarized and doubly refracted can be shown by similar experiments to those used for light, but with delicate thermometers in place of the eye, to ascertain the state of polarization or double refraction of the beam of heat. These facts show that radiant light and radiant heat must be of the same nature, and that radiant heat comes from the sun in about 8 minutes and 13 seconds, and moves with the velocity of 192,500 miles per second, in like manner with light.

In order to show that dark heat, or that which radiates from hot bodies unaccompanied by light, follows the same laws as that which accompanies light, we require sensitive methods of measuring temperatures, such as the differential and air-thermometers, and the thermo-electric multiplier of Nobili. The former instruments are sufficient to prove the laws of the reflexion and refraction of dark heat to be identical with those of light; and by means of the thermo-multiplier Professor Forbes and M. Melloni have proved its polarization and double refraction.

As the light from the sun consists of all the colours of the solar spectrum with their different degrees of refrangibility, so the heat which accompanies it has different degrees of refrangibility, but it passes through a plate of transparent glass equally with the light, which is shown by the burning glass. The heat, however, from a furnace or vessel of hot water, or from a ball of metal heated below redness, passes only in a very small degree through a plate of glass, and the glass becomes heated. This is

an effect similar to what we see with coloured glasses, which transmit some colours of light, and absorb the others. Rock-salt or chloride of sodium is found to transmit all kinds of radiant heat and light equally, or it has nearly perfect diathermancy as well as transparency. From this property prisms and lenses of rock-salt are important pieces of apparatus in the experiments upon dark heat.

When a thermometer is placed near a cold body it is found to fall, or there appears a radiation of cold rays. This arises from the thermometer giving off more radiant heat than it receives in return, and so becomes cooled. We must consider bodies to be always giving off radiant heat, and receiving it also from other bodies. If they receive more than they give off their temperature rises, and if less it falls, and if they receive heat equal to what they lose their temperature is stationary.

The conducting powers of bodies for heat can be compared by taking equal rods of them, inserting one end in a vessel in which water can be kept boiling, as at A, fig. 54, and at the other end attaching balls with soft wax, or putting pieces of phosphorus at equal distances on the rods. Then the time occupied by the heat in passing from the hot water to the wax or phosphorus, so as to melt the one or inflame the other, shows the relative conducting powers of the rods. The results for the conducting powers of the metals show that for silver it is far the best, then for copper, then for many intermediate metals; and then in iron and platina it is small comparatively to silver.

Fig. 54.
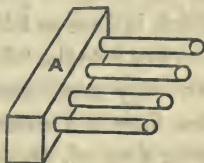
The liquids have been found to have very little, if any, conducting power for heat; for if heat be applied above still water or other liquid containing a thermometer, that instrument is so slightly affected that the result may be due to radiation. If, however, hot water is poured down a pipe so as to pass into cold water, the two mix, because the hot water is specifically lighter, and thus communicates its heat to the cold water by what is called *the convection* of heat. A hot body in the air cools by both radiation and convection.

On Thermometers.

In the uses of thermometers, which are very various, the instrument should be constructed according to the purpose to which it is to be applied. Thus, for meteorology, the scale should extend to the utmost limits of heat and cold which may be experienced in the climate of the place of observation; in other instruments, the accurate graduation about the temperature of boiling water may be most desirable; whilst in others, again, the most complete range of the scale may be needed. The best thermometers are supposed to accord with a carefully prepared standard thermometer, and to be graduated by comparison with such an instrument.

PROP. 47. *To explain the construction of a standard mercurial thermometer.*

A tube of glass of proper length and bore being found, the first thing is to determine its caliber in different parts of the bore, which is done as follows: pass some mercury into the tube as *AB*, fig. 55,

Fig. 55.
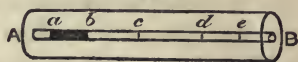
to occupy some convenient space as *ab*, which should be noted; then force the mercury forward until it occupies another portion as *bc*, then *cd*, *de*, &c.; which being all registered, when the instrument is graduated these spaces must each contain the same number of degrees, since they are of equal volumes.

The end of the tube being melted with the blowpipe, a bulb as *A*, fig. 56, is blown upon it, and often drawn into a phial shape to procure sensitiveness. The skill of the artist enables him to regulate the capacity of the bulb to that of the tube, so that the required range of scale may be obtained. The bulb and tube are then filled with mercury, which requires some dexterity of manipulation, in order that no particle of air may remain in the bulb. Then the bulb being heated up to the highest temperature for which it is required

Fig. 56.
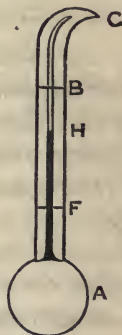
beyond the boiling point of water, the mercury filling the tube, the end *c* is closed by melting with the blowpipe, and often finished by bending, as in the figure, or formed with an indented ring round it.

The graduation is obtained by immersing the ball and stem in melting snow, noting the height of the mercury in the stem, say at *F* in the figure, called the freezing point; then placing them in the steam of boiling water when the barometer stands at 30 inches, as the average height at the level of the sea; let *B* be the place the mercury reaches to, which is called the boiling point. It is finally required to divide the interval *FB* into the number of degrees between the freezing and boiling points, allowing for the varying caliber of the tube; and the expansion of the mercury compared with that of the glass being considered uniform.

Fahrenheit considered that the mixture of snow and common salt produced the greatest degree of cold to be obtained, and marked that temperature the zero of his scale.

In Fahrenheit's scale the freezing point is marked 32°, and the boiling point 212°.

In Celsius's, or the centigrade scale, the freezing point is marked 0°, and the boiling point 100°.

In Reaumur's, the freezing point is marked 0°, and the boiling point 80°.

From the ascertained graduation between *F* and *B* the scale is continued above and below those points. In the best standard thermometers the degrees are etched upon the glass stem by fluoric acid. The thermometers filled with coloured spirit of wine cannot be employed at temperatures above that at which the spirit boils, but they have the advantage of showing the temperatures below the point when mercury congeals, as pure alcohol has never yet been solidified. They are graduated by comparing them with a standard mercurial thermometer, like the meteorological thermometers.

PROP. 48. *To investigate formulæ for comparing the corresponding degrees in Fahrenheit's, Celsius's, and Reaumur's thermometers.*

In figure 56, let H be the height the mercury stands in the stem at any time, F the freezing point, and B the boiling point. Let F^0 be the degrees on Fahrenheit's scale for the point H, C^0 those on the centigrade scale, and R^0 those on Reaumur's scale. Then we have the following ratios:

$$\frac{\text{space } FH}{\text{space } FB} = \frac{F^0 - 32^0}{180^0} = \frac{C^0}{100} = \frac{R^0}{80^0},$$

$$\text{or } \frac{F^0 - 32^0}{9} = \frac{C^0}{5} = \frac{R^0}{4}.$$

From which, when any one of the three quantities, F^0, C^0, R^0 is given, the others may be found. Fahrenheit's scale has the advantage of not requiring the mention of negative degrees in ordinary atmospheric temperatures.

Ex. 1. When the temperature is 60^0 in London, what would be the degrees named in Paris and Vienna?

Then $F^0 = 60^0$. The C^0 named in Paris would be $15^0\frac{5}{9}$, and the R^0 in Vienna would be $12^0\frac{4}{9}$.

Ex. 2. When the temperature is 20^0 in London, what would it be called in Paris and Vienna?

$$\frac{0-32}{9} = -\frac{12}{9} = -\frac{4}{3} = \frac{C^0}{5} = \frac{R^0}{4} \quad \therefore \quad C = -6\frac{2}{3} \quad R = -5\frac{1}{3}$$

PROP. 49. *To explain the construction and properties of an air-thermometer.*

A tube of glass with a bulb A, fig. 57, blown on the end of it, having its open end placed in some vessel of liquid, such as coloured water, and being supported by a cork through which it passes in the neck of the vessel, will form an air-thermometer for comparative experiments. Let BC be the level of the surface of the liquid in the vessel, then applying heat to the bulb A, some of the contained air will pass out of the open end through the liquid, and when the instrument is again cooled to the temperature of the atmosphere, the liquid in the tube will stand at some point as a in the figure. Graduation may be obtained by taking the instrument into a cold place of known tem-

Fig. 57.
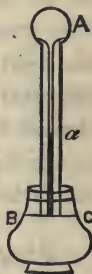
perature, and then into a hot one of known temperature; and having noted the heights of the liquid in the two cases, dividing the interval into the requisite degrees, and noting at the same time the height of the barometer.

The air-thermometer, when constructed for accurate measures, is on a like principle to the above, but requires many precautions; and, being affected by the atmospheric pressure, can only be used with certainty when the barometer is stationary, unless laborious calculations are submitted to. The liquid used must be mercury, so that vapour in the bulb may not disturb the results, and the air or gas filling it must be dried at commencing the construction. In reading the scale the liquid inside and outside the tube must be brought to the same level, after allowing for the capillary depression of the mercury in the tube; and this requires a high vessel to hold the mercury.

PROP. 50. *To explain the construction of the differential air-thermometer.*

The differential thermometer has two bulbs as *A* and *B*, fig. 58, connected by a tube, and before the opening to the atmosphere is closed, some coloured sulphuric acid is passed into the tube to occupy some part as *acb*, and then the connexion with the external air is closed with the blowpipe. When the two bulbs *A* and *B* are equally heated, the terminations *a* and *b* of the sulphuric acid remain at rest; but if one bulb is heated more than the other, the increased elastice force of the air in it drives the liquid *acb* towards the other bulb. The instrument in this manner shows differences of temperatures, and the bulbs may be at the same height, or at different heights.

Fig. 58.
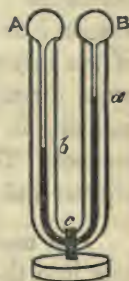
PROP. 51. *To explain the construction of self-registering thermometers.*

The self-registering thermometers of Dr Rutherford and Mr Six have been the most used. The former will be here described. The most complete self-registering meteorological

and magnetic instruments are those of Mr Brooke, acting by photography with gas-light and taking continuous results, but requiring to be attended to frequently.

In the figure 59 a mercurial thermometer has its stem horizontal, which being of a wide bore, a short length of a steel needle as a is placed within it. As steel does not amalgamate with mercury, the latter, as it advances, pushes the

Fig. 59.

needle a before it and leaves it at the highest point it reaches, and marks the maximum temperature which has occurred.

In the figure 60 a spirit-of-wine thermometer, with its stem horizontal, has a short length b of black glass or enamel inside the spirit. The attraction of the spirit for the enamel causes it to be carried back as the temperature falls, and it is

Fig. 60.

left at the lowest temperature which occurs, and thus marks the minimum. After noting the results, the observer brings the enamel b to the end of the spirit in the stem by raising the end of the instrument which has the bulb of the spirit for the minimum thermometer, and brings the steel needle a to touch the mercury in the maximum thermometer by means of a small magnet. The instruments are then prepared to register the highest and lowest temperatures in another interval of time.
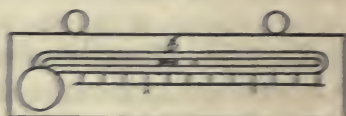
The maximum instrument requires care, for if the steel a gets into the mercury it becomes soiled, and must be taken out and cleaned before the instrument will act well again.

The minimum instrument is not liable to get out of order, for the enamel b does not easily get out of the spirit; and if it should get out, it is easily brought to its place again inside the spirit.

Thermometers are sometimes made advantageously of solid materials entirely. Wedgewood's pyrometer consisted of pieces of porcelain clay of known size, which being submitted to the

heat of a furnace were found to contract in bulk according to the heat to which they had been exposed; and thus furnished a means of finding the temperature. The indications of this instrument were, however, uncertain, from the same effect being produced by a high temperature acting for a short time, as by a lower temperature acting for a longer time.

Ferguson's pyrometer is more adapted to compare the expansions of different substances than to ascertain temperatures; but Breguet's thermometer and Daniell's pyrometer are valuable instruments in particular cases for determining temperatures, better than any others for those cases.

PROP. 52. *To explain the construction of Ferguson's pyrometer.*

A bar of metal, or other substance, as AB, fig. 61, is placed between the ends of two rods of glass; the end A of one rod being screwed up to its place and stationary, the other Bb is free to move in its support when the bar AB expands on being heated in a bath of hot water or otherwise. The end b of the rod Bb presses at b near to C on the lever Ca, which turns about

Fig. 61.
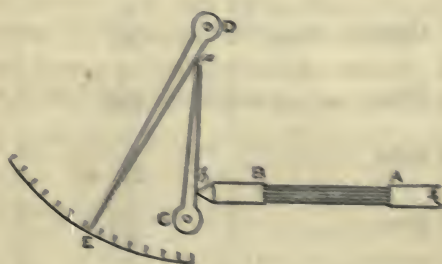
a pivot at C, and this lever Ca presses at a against another lever DE near its fulcrum or center D. The effect of the expansion is thus magnified, and the point E of the second lever acts as an index on a graduated arc, and by its motion along the arc shows the expansion of the rod AB, when the mechanical advantages

of the levers have been found, and the effect of the heat upon the glass rods. By placing different rods of metals at *AB* their expansions can be found.

There have been many other methods employed to determine the expansions of bodies. The following tables contain the expansions of a few principal substances.

Substance.	Expands between 32° and 212°.	Expands between 212° and 392°.	Expands between 392° and 572°.
Mercury	$\dfrac{1}{55\cdot50}$	$\dfrac{1}{54\cdot25}$	$\dfrac{1}{53\cdot00}$
Glass	$\dfrac{1}{387\cdot00}$	$\dfrac{1}{363\cdot00}$	$\dfrac{1}{329\cdot00}$
Mercury in glass......	$\dfrac{1}{64\cdot80}$	$\dfrac{1}{63\cdot78}$	$\dfrac{1}{63\cdot18}$

Substance.	Expands between 32° and 212°.
Silver	$\dfrac{1}{174}$
Copper	$\dfrac{1}{194}$
Brass	$\dfrac{1}{185}$
Gold	$\dfrac{1}{224}$
Iron	$\dfrac{1}{282}$
Platina............	$\dfrac{1}{377}$

From the latter table it will be seen that when a thin strip of brass is rivetted or brazed to a like strip of iron, then if the compound bar is straight at one temperature, it will not be so at

other temperatures. In fig. 62, if the bar were straight as in the upper figure, it would become convex on the side of the brass at higher temperatures as in the lower figure, but concave at lower ones. Such compound bars have been used to produce

Fig. 62.

thermometers, self-regulating shutters to buildings, &c.; but their most important use is in chronometers, where such small compound strips of brass and steel used in the construction of the compensation balance-wheel counteract the effect of change of temperature on the balance-spring, and so the chronometer maintains the same rate of going at different temperatures.
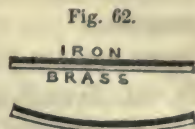
When the expansion of a body in volume is known, as in the above tables, the linear expansion is found by a simple rule, as in the next proposition; and the converse.

PROP. 53. *To show that the expansion of a body in volume is three times its linear expansion, nearly.*

Let V be the original volume, and l the distance of any two points in it.

Let V' be the volume when the distance of the same points is $l + x$, and x is very small;

$$\text{then} \quad \frac{V'}{V} = \frac{(l + x)^3}{l^3};$$

therefore the expansion in volume

$$= \frac{V' - V}{V}$$

$$= \frac{(l + x)^3 - l^3}{l^3}$$

$$= \frac{3x}{l} + \frac{3x^2}{l^2} + \frac{x^3}{l^3}$$

$$= 3\,\frac{x}{l}, \text{ nearly, since } x \text{ is very small,}$$

$$= 3 \times \text{linear expansion.}$$

PROP. 54. *To explain the construction of Breguet's thermometer.*

The essential part of Breguet's thermometer is a spiral AB, fig. 63, formed of a compound flattened wire consisting of silver, gold, and platina, with the gold between the silver and platina, in accordance with its expansibility, as seen in the table. The compound wire is rolled to be very light and thin, and there are twenty-three revolutions in the spiral of the instrument. The end A is fixed by a piece of brass with a screw to a part AC, which is supported by the brass arm CD, but can be turned round to procure adjust-

Fig. 63.

ment of the index ab at the other end of the spiral. The index ab is a light metallic index fastened to a needle soldered to the lower free end of the spiral, and moves horizontally, with variation of the temperature, over a divided circle on the base or pedestal of the instrument.
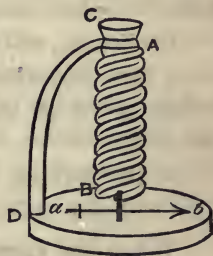
The advantage of this instrument is the rapidity, almost instantaneous, with which it shows the temperature of the air in which it is placed, and can be thus used where the sluggishness of the other thermometers is a serious objection.

PROP. 55. *To explain the construction of Daniell's pyrometer.*

Daniell's pyrometer consists of a rectangular rod of plumbago AB, fig. 64, with a hole down its axis to receive a rod ab of thick iron or platina wire. The end a of the rod ab resting against the bottom of the hole, a piece of porcelain or tobacco-pipe bc is pushed against the other end b of the rod, and is prevented slipping easily by a band of platina, with a tightening wedge of porcelain, passing round it and a projecting part of the plumbago.

Fig. 64.
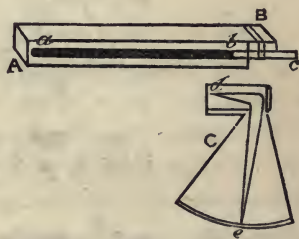
The difference of the expansions of the plumbago and metallic rod *ab* is ascertained by an instrument like *C* in the figure, which has a lever turning about a pivot with the end *e* as index moving along a graduated arc, and an edge at *d* to be applied to a notch at *c* on the end of the porcelain rod, when the instrument *C* is put in its place at the end of the rod of plumbago.

The values of the divisions on the graduated arc can be found by comparison with a mercurial thermometer below the boiling point of mercury, and then extended to higher temperatures.

As the materials of which the pyrometer is constructed bear very high temperatures unchanged, the temperatures of furnaces and the melting points of various metals are determined by it with considerable accuracy.

Below is Professor Daniell's table of these temperatures.

Metals.	Fusing points.
Tin	442°
Lead...............	612
Zinc	773
Silver	1873
Copper	1996
Gold	2016
Cast iron	2786

On the Specific Heat of Bodies.

It has been explained that by the specific heats of bodies we mean the relative amounts of caloric which they require to be communicated to or taken away from them to raise or depress them respectively a given number of degrees of temperature, and that their capacities for caloric are supposed to be represented by the same numbers as their specific heats, when the same standard is adopted.

Let c = the capacity for caloric of a body, or the amount of caloric required to raise a unit of mass *one* degree of temperature.

Then if c be constant, the amount required to raise a mass m one degree will be mc; and to raise the mass m, t degrees of temperature will be $m \cdot c \cdot t$.

Let c', m', t' be like quantities for another body, then the amount of caloric required to raise or depress it t' degrees will be in like manner $m' \cdot c' \cdot t'$.

When these amounts are equal, we have

$$m \cdot c \cdot t = m' \cdot c' \cdot t',$$

$$\text{or } \frac{c}{c'} = \frac{m' \cdot t'}{m \cdot t};$$

$$\text{and if } m = m', \text{ then } \frac{c}{c'} = \frac{t'}{t}.$$

Suppose any weight, as one pound of iron, to be quickly transferred from boiling water into an equal weight of water at $56°$, then they will soon have acquired a common temperature of about $72°$, or the caloric which iron has given out in falling $212° - 72° = 140°$, has raised the equal weight of water $72° - 56° = 16°$.

Therefore if c' the capacity of water be taken unity, we have c the capacity of iron from the formula

$$c = c' \frac{t'}{t} = 1 \times \frac{16}{140} = \cdot 114,$$

or the capacity of iron is about $\frac{1}{9}$th that of water for equal weights.

To obtain the capacity for equal bulks we must multiply this by the specific gravity of iron, say $7 \cdot 8$, and have the capacity of iron $= \cdot 8892$; or in equal bulks iron contains about $\frac{9}{10}$ths the caloric which water contains. If the iron were at any lower temperature than $212°$ the value of the capacity would be found to be the same, unless exceedingly great accuracy of manipulation and observation is used.

The capacities of solid bodies generally are easily found by this method of immersion, but in practice several precautions are required, in order to obtain exact results; for the effect of the vessel in which the water is contained must be ascertained and allowed for, and also care must be taken to avoid loss of heat in the manipulation.

If equal weights of water or of mercury at different temperatures are mixed, the resulting temperature is very nearly the mean; but the careful experiments of MM. Dulong and Petit showed that the capacities of the metals increase sensibly with the temperatures; and the same property has been found to hold generally with liquids and gases, when they have been very carefully examined.

There have been other methods employed to determine the specific heats of bodies, which give nearly the same results as that of immersion: one of these is by measuring the quantity of ice at the freezing point, which is melted by a body of given weight and temperature in an instrument called a calorimeter; and another is by noting the time occupied in the cooling of the bodies from given temperatures. No method of mixing together liquids which act chemically upon each other can of course be applicable to determine their specific heats.

	Specific heats for equal weights.
Water	1·000
Iron	·114
Copper	·096
Zinc	·094
Arsenic	·081
Silver	·059
Tin	·056
Cadmium	·057
Antimony	·052
Gold coin	·034
Bismuth	·033
Platina............	·033
Mercury	·033
Lead...............	·032
Sulphur	·190
Flint glass	·190

The determination of the specific heats of the gases and vapours is a subject of great importance, and was not accurately

accomplished until MM. Delaroche and Berard undertook the investigation. By passing the heated gases through a long spiral tube in a vessel of water, and observing the temperature at entering the tube in the water and at leaving it, together with the heat communicated to the water, they obtained the results in the table below.

| | Specific Heats. | |
	For equal bulks.	For equal weights.
Air	1·0000	1·0000
Hydrogen gas ...	·9033	12·3401
Carbonic acid ...	1·2583	·8280
Oxygen............	·9765	·8848
Nitrogen	1·0000	1·0318
Nitrous oxide ...	1·3503	·8878
Olefiant gas	1·5530	1·5763
Carbonic oxide...	1·0340	1·0805
Aqueous vapour.	1·9600	3·1360

When water is taken as the standard, the results become as in the next table.

	Specific heats for equal weights.
Water	1·0000
Air	·2669
Hydrogen gas ...	3·2936
Carbonic acid ...	·2210
Oxygen............	·2361
Nitrogen	·2754
Nitrous oxide ...	·2369
Olefiant gas	·4207
Carbonic oxide...	·2884
Aqueous vapour.	·8474

Upon these results, Dalton, in the Appendix to the first part of Volume II. of his *New System of Chemical Philosophy*, has the following important remarks: "From the foregoing detail of experiments on elastic fluids, it appears evident that such fluids exhibit matter under a form in which it has the greatest possible capacity for heat, when capacity is understood to denote the total quantity of heat connected with the fluid; but if the capacity or specific heat is meant to denote the quantity of heat necessary to raise the body a given number of degrees of temperature, then the elastic-fluid form of matter is that which has the least capacity for heat of any known form of the same matter. When therefore we use the term *specific heat* as applied to elastic fluids, we should henceforward carefully distinguish in what sense they are used; but the terms may still be indifferently used in the one or the other sense as applied to liquids and solids, till some more decisive experiments show that a distinction is required. Probably the anomalies that have occurred in investigating of the zero of cold, or point of total privation of heat, are in part due to the want of accordance between the ratio of the total quantities of heat in bodies, and the ratio of the quantities producing equal increments of temperature," &c.

When the gases are suddenly condensed in the fire-syringe it was stated that the heat produced is often sufficient to fire tinder. This arises from the condensed gas having less capacity for caloric than it had before the condensation, so that the temperature is raised by the condensation. The law of this change of capacity for caloric in gases is of importance in the theory of sound, and some experiments seem to show that the instantaneous change, as found in experiments by Mr Joule and the author, may be different from that which takes place after some interval of time, when the caloric combined with the dense matter has attained a new statical condition.

ON VAPOURS.

It has been stated before that the vapours differ from the gases only in being easily reduced to liquids by cold or pressure, or both. Whilst the vapour retains its elastic state it is subject to the same laws as the gases, that is, Boyle's, Gay Lussac's, and Amonton's laws apply to them, at temperatures which are distant from their dew-points or points of liquefaction, but are found to fail in accuracy near those points: there can be little doubt but that the gases are subject to like failures near their points of liquefaction.

When a vessel of any liquid is placed under the exhausted receiver of an air-pump an amount of the liquid rises in vapour which depends upon the temperature. When the evaporation ceases the space in the receiver is said to be saturated with the vapour, which thus attains a certain degree of elastic force and density varying with the liquid and the temperature. When such a space thus saturated with vapour is reduced in temperature, a portion of the vapour becomes liquid again, and is exhibited in mist through it, or as dew on the vessel; and so also if it is subjected to additional pressure and allowed to return to its original temperature.

It is found that, if the vessel of liquid were placed under the same receiver filled with dry air or gas, the same amount of liquid would rise in vapour as in the exhausted receiver, only more slowly, and the elastic force of the saturated air or gas would be the elastic force of the dry air or gas plus that of the vapour.

Let p be the pressure on a unit of area due to the elastic force of the dry air or gas;

f be the pressure on a unit of area due to the elastic force of the vapour;

p' be the pressure on a unit of area due to the elastic force of the saturated air or gas;

then $p' = p + f$ when the volume is that of the original dry air or gas, and if any two of the quantities p, p', f are given, the remaining one is known; but f will be known from tables of the elastic force of vapours at different temperatures, and therefore $p = p' - f$ for air or gas if dry will be known.

Secondly, let the pressure remain the same, and let V' be the volume of the saturated air or gas when the pressure is p, V the volume the dry air or gas would occupy at the same pressure.

Then, by Boyle's law,

$$\frac{\text{elastic force of the air or gas in volume } V'}{\text{elastic force of the air or gas in volume } V} = \frac{V}{V'} = \frac{p-f}{p},$$

$$\text{and } V = V'\left(\frac{p-f}{p}\right), \quad \text{or } V' = V\left(\frac{p}{p-f}\right),$$

which give either V or V' when the other with p and f are known.

That solids may furnish vapours as well as liquids is shown in the case of ice and snow, which may be easily noticed to diminish during a long frost; and such substances as camphor disappear in vapour quickly when exposed to the air: so that steam or aqueous vapour exists of an elastic force, which can be measured far below the zero of Fahrenheit's scale, as has been shown by M. Regnault. The subject of the elastic force of steam at different temperatures is so important that many philosophers have directed their attention to it; and a collection of their results will be found in the *Philosophical Magazine* for January 1849, in a reprint of a paper by J. H. Alexander, Esq.

As containing results for the vapours of other liquids as well as water, a table of some of Dalton's results is inserted here.

Temperature by common Thermometer.	Elastic force of the Vapours in inches of Mercury.				
	Ether.	Sulphuret of Carbon.	Alcohol.	Acetic Acid.	Water.
7°	3·75	3·134	·193		·11
35	7·5	6·20	·560	·27	·29
65	15	12·26	1·51	·69	·75
97	30	24·26	4·07	1·77	1·95
133	60	48·	11·00	4·54	5·07
173	120		29·70	11·7	13·18
220	240		80·2	30·	34·2
272					88·9
340					231·

It is evident that the elastic force of vapours increases in a very high ratio to the temperatures. Many attempts were made to find the relation between them, but we must not expect greater conformity between calculation and experiment than is shown by the formula of Mr Alexander, which does not differ, in the whole range of temperatures which have been investigated, more from the experiments of the different investigators than they do from each other.

Mr Alexander's formula is

$$p = \left\{ \frac{t}{180} + \frac{990}{1695} \right\}^6,$$

where p is the pressure due to the elastic force of *steam* in inches of mercury, and t is the temperature on Fahrenheit's scale. The following table contains a few selected results from those occupying four pages of Mr Alexander's paper.

Temperature Fahrenheit.	Pressure in inches of Mercury of Steam.		
	By the formula.	By observation.	Observers.
− 27°·112	·0066	·0106	Regnault.
− 13·	·0180	·0205	......
− 4·504	·0305	·0284	......
+ 1·706	·0437	·0457	......
9·41	·0664	·0638	......
32	·1956	·1811	......
40	·275	·250	Ure.
70	·849	·726	......
80	1·184	1·010	......
100	2·192	1·860	......
130	4·969	4·366	......
160	10·214	9·600	......
173	13·612	13·18	Dalton.
222	34·73	34·20	......
		34·95	Taylor.
250	58·99	59·12	......
		61·19	Ure.
300	130·02	139·70	......
		133·75	Taylor.
340	228·74	231·00	Dalton.
372	346·95	325·	Arzberger.
403·88	511·43	514·22	French Acade-
435·227	731·92	716·13	micians.

The specific heat of steam at different temperatures is a subject of great importance in the theory of the steam-engine. Mr Watt found that the heat which became latent at the boiling temperature when water was converted into steam at the same temperature was sufficient to have raised the water 950° Fahrenheit; and by Mr Southern's experiments it is nearly the same in steam at different temperatures and degrees of elastic force. This shows that when steam is condensed into water

again by cold applied to it, it gives out heat which would raise an equal weight of water 950° of temperature.

The boiling points of liquids are the temperatures at which the elastic force of their vapours equals the pressure to which they are subject, and thus becomes lower as the pressure is less.

The following table from Dalton's *Meteorological Essays* gives the boiling temperatures of water under different pressures of air in the receiver of an air-pump.

Heat of the water when boiling.	Pressure upon its surface in inches of Mercury.	Rarefaction of the air.
212°	30·0	1·
200	22·8	1·3
190	18·6	1·6
180	15·2	2·0
170	12·2	2·45
160	9·45	3·2
150	7·48	4·0
140	5·85	5·1
130	4·42	6·8
120	3·27	9·2
110	2·52	11·9
100	1·97	15·2
90	1·47	20·4
80	1·03	29·0

M. De Saussure found the heat of boiling water upon the summit of *Mont Blanc*, 186°; the height of the mountain is near three miles above the level of the sea; the barometer was 16 inches $\frac{144}{160}$ of a line (a little above 17 English inches).

Dr Wollaston proposed to use the thermometer, graduated accurately with large divisions about the boiling point, to determine the heights of mountains. Such an instrument is much more portable than the barometer, and a small quantity of water can be boiled by spirit of wine very quickly and easily in the vessels to be used with the thermometer.

In determining the elastic force of the vapour in contact with the liquid from which it arises, there are two cases, as the elastic force of the vapour is less or greater than that of the atmosphere.

PROP. 56. *To explain the method of determining the elastic force of a vapour when it is less than that of the atmosphere.*

Barometer tubes and well boiled mercury being prepared, let a tube filled with mercury, and freed from all air-bubbles, have its open end covered until it is placed below the surface AB of the mercury in the cup, fig. 65, and it will then be a barometer, with the upper surface of the mercury in the tube resting at some point a, as in the figure. Let another tube be filled in the same way, except a small portion at the open end, which being filled with the liquid of which the vapour is to be examined, let the end be then closed with the thumb until that end is below the surface AB of the mercury in the cup. The liquid being specifically lighter than mercury rises through it to the upper closed end of the tube, and the tube being vertical, when the thumb is withdrawn the mercury falls to some point as c with a portion bc of the liquid resting upon it. The portion of the tube above b is filled with the vapour of the liquid, and its elastic force is measured by the column of mercury whose height is the difference of the heights of a and c above the level of AB. The liquid bc and the space above it being brought to a variety of temperatures, the elastic force of the vapour becomes known for those temperatures, as long as it does not exceed the pressure of the atmosphere when the point c has come down to the level of AB.

Fig. 65.

PROP. 57. *To explain the methods of determining the elastic force of a vapour when greater than that of the atmosphere.*

Figures 66 and 67 represent the two instruments of the methods of determining the elastic force of vapours when greater than that of the atmosphere.

ABC, fig. 66, represents a bent tube of glass, closed at the
end *A*, and open at the end *C*.
Mercury being passed into the tube
to fill the leg *AB* and part of *BC*,
some of the liquid whose vapour is
to be examined is passed through
the mercury to occupy a small por-
tion of the tube near *A* when placed
vertical. Heat being applied to the
liquid near *A* by a vessel of heat-
ed oil surrounding it, or otherwise,
when vapour is formed above the
liquid let the surfaces of the mercury
in the two legs be *a* and *b*. Draw
a horizontal line from *a* to *a'*, then
the elastic force of the vapour at *A* supports the column of mer-
cury *ba'* together with the pressure of the atmosphere; and
the elastic force is expressed in inches of mercury by the height
of the barometer plus the height *a'b*.
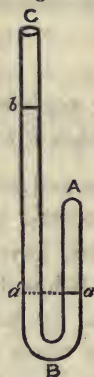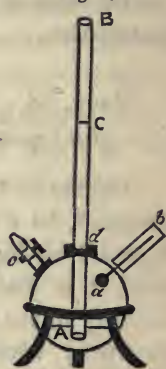
Figure 67 represents Marcet's boiler, which consists of a
strong spherical metal vessel supported on a tripod. It has
three apertures to which are adapted, by screws and steam-tight,
first, a thermometer *a b*, with its bulb inside the boiler to show
the temperature of the vapour; secondly, an aperture with a
stuffing-box at *d* to admit a long straight tube of glass, open at
both ends as *BA*; and thirdly, an aperture with a pipe and
stop-cock *c* as in the figure. In using the instrument, a known
quantity of mercury is poured into the boiler, and the lower end
of the tube of glass is passed below its surface *A* in the figure.
A quantity of the liquid whose vapour is to be examined is then
poured upon the mercury, when the thermometer and pipe with
the stop-cock are screwed in their places. Now, the heat of a
lamp being applied under the boiler, when the liquid boils, if
the stop-cock *c* is left open, the atmospheric air in the boiler
will be forced out by the ascent of the vapour, and when the
vapour only issues through it the stop-cock can be closed, and,
the heat being still applied, the temperature of the vapour is
shown by the thermometer *a b*, and its elastic force by the

height, as AC in the figure, to which the mercury is forced in the glass tube AB. The elastic force of the vapour at the temperature shown is measured, in inches of mercury, by the height of the barometer plus the height of the column AC.

If the stop-cock c is opened when the elastic force of the vapour is considerable, the hand may be held near the aperture and will only feel a cold jet of condensed vapour to strike it. This arises from the sensible heat of the vapour having become latent during the expansion of the jet after issuing into the air. It is thus sometimes said that high-pressure steam blows cold, whilst a jet of low-pressure steam will scald severely.

PROP. 58.   *To explain the mode of action of Dr Wollaston's instrument as an elementary steam-engine.*

This instrument consists of a cylindrical tube of glass AB, with a bulb BC, blown on one end of it, as in fig. 68.   In the cylinder there moves a steam-tight piston acb, with its piston-rod ced, a tube which can be closed at the end d, by screwing on the cap d, as in the figure ; it passes loosely through an aperture e in the brass cover to the end of the cylinder A, which is connected with a handle f. If the bulb BC be filled with water, and the piston be then pushed into the tube with the end d open, the air will pass out through the tubular

Fig. 68.

piston-rod ced, and when the piston is pushed to the surface of the water the cap d may be screwed on, and so as to close the end at d. If the bulb BC be now held over the flame of a lamp, when the water in BC comes to the boiling temperature, and the elastic force of the steam equals the atmospheric pressure, the steam will rise and fill the cylinder AB, forcing the piston to the top.   If the instrument be now removed from the flame the heat will pass away by radiation and the convection of the air, and the steam in AB will condense into water, when the pressure of the atmosphere on the upper side of the piston will force it down again ; and the same process may be repeated.
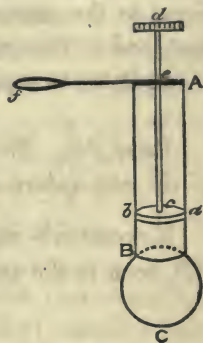
If we consider this experiment in its simplest form with M. Carnot, we consider a cylindrical tube with tight piston closed at one end and open at the other, the piston being at the closed end, but with a quantity of water at the end, which, when converted into steam at the atmospheric pressure, will fill a given volume of the cylinder. Let sufficient heat be now communicated gradually to the water to convert it into steam, the piston will move up the cylinder until it comes to its position of equilibrium, with the pressures on each side of it equal. If the heat be now abstracted again the piston will return to its first position, and then the same operations may be repeated continuously. In these operations the work done, neglecting friction, momentum of the piston, &c. is measured by the quantity of air forced from the cylinder, and this equals the quantity of steam at the atmospheric pressure which is formed by the heat communicated and abstracted; which ought to be proportional to the amount of fuel consumed in order to produce that heat, and to the quantity of oxygen gas of the atmosphere which is employed to support the combustion.

PROP. 59. *To explain the construction and mode of action of the atmospheric pumping engine.*

Newcomen's atmospheric steam-engine had an iron cylinder *AB*, open to the atmosphere at the top, but closed at the bottom

Fig. 69.
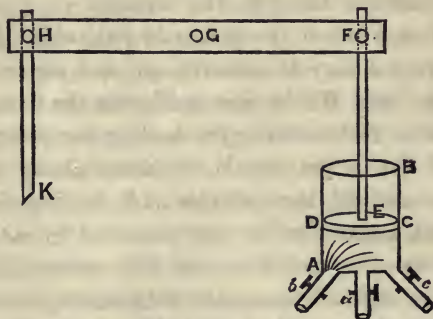
except where three pipes entered, called the steam-pipe, the cold-

water pipe, and the condensed water-pipe respectively, as in fig. 69. The pipes had each a stop-cock, which could be opened and shut by an attendant. The cylinder had an air-tight piston *CD* within it, and the piston-rod *EF* was connected with the beam turning about an axis *G*. The pump-rods, as *HK*, were connected with the other end of the beam. Let *a* represent the stop-cock in the steam-pipe coming from the boiler, *b* that in the cold-water pipe coming from an elevated cistern, and *c* that in the condensed water-pipe.

Then if the piston *CD* were at the bottom of the cylinder and the stop-cock *a* were opened, the steam from the boiler would enter the cylinder and the pump-rods preponderating the piston would rise to the top of the cylinder; the stop-cock *a* being then shut and *b* opened a jet of cold water entering the cylinder, as in the figure, the steam would be condensed and a vacuum formed under the piston, when the pressure of the atmosphere being nearly 15 pounds on each square inch of the area of the piston, it would be forced down and raise the pump-rods at the other end of the beam. The stop-cock *b* being closed when the condensation was complete, that at *c* would be opened to allow the water from the injection and condensed steam to escape from the cylinder, and then *c* being closed, *a* would be opened again and another stroke of the piston take place, and so onwards.

The disadvantages of this steam-engine were, that when the cylinder and piston were cold there was a loss of steam, which entered the cylinder when *a* was opened, by condensation, and, on the other hand, when they were heated the cold water injected into the cylinder did not completely condense the steam to produce a vacuum. Mr Watt's improvements were first directed to remedy these defects by performing the condensation in a separate vessel, and keeping the cylinder always heated, and afterwards he contrived both single- and double-acting engines with closed cylinders.

PROP. 60. *To explain the principle of the double-acting condensing steam-engine.*

Let *ABCD* represent the cast-iron cylinder of the engine, *EF* the induction-pipe or steam-pipe from the boiler, *GH* the eduction-pipe leading to the condenser *K*, into which enters the cold-water pipe *k*, and from which a pipe *l* leads to the air-pump of the engine. Let *abcd* represent valves opening into the cylinder, which can be opened and shut; *egf* the steam-tight piston, and *gh* the piston-rod passing through the steam-tight stuffing-box *i*.
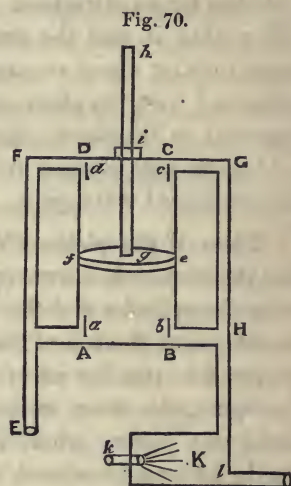
Suppose the cylinder to be heated and filled with steam, by the operation called 'blowing through' by the engineers, then if the valves *a* and *c* be opened the steam above the piston rushing through *c* to the condenser and meeting the jet of cold water from the pipe *k*, will be condensed, and there will be a vacuum above the piston and the pressure of the steam below, which entering by *a* forces the piston to the top of the cylinder, when the valves *a* and *c* are shut and those at *d* and *b* are opened. The steam below the piston now rushes through *b* and becomes condensed, leaving a vacuum below the piston; and the steam from the induction-pipe entering by *d* forces the piston to the bottom of the cylinder, and then the process goes on as before. The reciprocating motion of the piston is communicated to the main beam of the engine by the piston-rod, and from it the connecting rod goes to the fly-wheel, which produces an equalized rotatory motion by its moment of inertia.

The figure 70 being only for illustration and explanation the practical arrangement of the valves would always be very different, and there have been numerous different constructions used, as the *D* valves of Bolton and Watt, the box-valves, the plug-valves, the three-way cock, &c.

Fig. 70.

PROP. 61. *To explain the mode of action of the box-valve of a steam-engine.*

Let *A* be the induction-pipe, *BC* a chamber into which it leads, having three pipes from it, as in fig. 71, of which *a* leads to the bottom of the cylinder, *d* to the top, and *c* leads to the eduction-pipe and condenser. A box *be* slides upon the smooth face of the chamber *BC* into which the three pipes open, and is sufficiently long to cover two of them only at the same time. It is moved up and down by the rod *ef* passing through a stuffing-box, and connected with the engine. In the position of the box *be*, as in the figure, we see that the induction-pipe is in communication through *a* with the lower part of the cylinder, and that the upper part is in communication through *d* and *c* with the condenser.

Fig. 71.
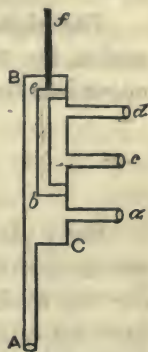
When the box *eb* is pushed down to cover the openings of *c* and *a*, we see that the induction-pipe will be in communication with the upper part of the cylinder, whilst the lower part will be in communication with the condenser. In this way the action of the four valves of fig. 70 can be performed by the sliding up and down of a box like *be*, fig. 71, which is kept close to the face of the chamber by having a vacuum on its inside and a pressure of steam on its outside.

PROP. 62. *To explain the construction of the high-pressure steam-engine.*

The condensing engine will evidently work with steam of the elastic force of the atmosphere. Mr. Trevithick saw that by using high-pressure steam a much simpler engine could be employed, dispensing with the condenser, air-pump, &c., and the loss of power which was thus avoided might nearly compensate for the want of condensation. A sufficient supply of cold condensing water is also frequently difficult to obtain. In such cases Mr Watt's condensing engines would be of little value, and could not, for instance, be used on railways. The high-pressure engines are now exceedingly numerous; and when

Metropolitan Railway?

high-pressure steam super-heated, with condensation, and when the entrance of the steam into the cylinder is cut off at part, say one half or one third the stroke, and then acts expansively by its elastic force, the engine becomes the most economical of all.

The high-pressure engine is constructed like the condensing engine, fig. 70, but without a condenser, and the eduction-pipe opens into the air, or into the chimney of the furnace.

Many plans of rotatory steam-engines have been invented to produce rotatory motion at once, but from the difficulty of keeping the working parts steam-tight in such arrangements, they have not established themselves, although they promised at the first sight a great saving of power.

The steam-hammer consists of a heavy hammer or ram, which is raised by high-pressure steam acting upon the underside of a piston in a cylinder, and on the steam being allowed to escape the hammer falls directly upon the object to be operated upon.

On Hygrometers.

The hygrometer is an instrument for showing the degree of moisture or dryness of the atmosphere. There are very many substances which possess the property of being as it is termed *weatherwise*, or of being affected by the amount of vapour in the air where they are placed. Sponge, seaweed, hair, strips of whalebone, the awn of the wild oat and feathergrass, cords formed of animal and vegetable fibres, and many other substances have been used for hygrometers. Though very sensitive to changes of moisture the hygrometers formed of these substances have the disadvantage of not furnishing a scale which can be compared at distant times. Dalton observed for a long time the indications of a hygrometer formed with about 6 yards of whipcord fastened to a nail at one end, and thrown over

a small pulley, being stretched by a weight of 2 or 3 ounces at the free end. It had a scale divided into tenths of inches. In different states of the air in a room without fire but with a moderate circulation of air, it varied in length above 13 inches, being longer when the air was drier. It was found that the observations of different years could not be compared, as the cord continually increased in length with the time it was used. The hygrometers of like principle are liable to the same objection.

Leslie's hygrometer consisted of a differential thermometer, having one of its bulbs covered with thin cloth, which was always kept moist by a few threads leading to a vessel of water, and it showed the degree of evaporation from the cold produced by the indication of the thermometer. Daniell's hygrometer was used with æther, and required some attention.

The ascertaining the dew-point *directly* is a certain but laborious method of determining the moisture in the atmosphere. It consists of using cold water with freezing mixture in solution when necessary, which is poured from one glass vessel to another until dew is only perceptibly formed upon the glass, when the temperature of the water is noted. This temperature is more below that of the air as the air is the drier. This laborious method, which had been practised by some meteorologists for years, is now unnecessary, since the wet and dry bulb thermometer gives equally correct indications by inspection; and by means of Mr Glaisher's tables, given in his pamphlet on the instrument and its uses, the quantity of water in a given bulk of air is easily found, and thus a great service has been rendered to the science of meteorology by his investigations.

PROP. 63. *To explain the uses of the hygrometer called the wet and dry bulb thermometer.*

The wet and dry bulb thermometer, as in fig. 72, has two mercurial thermometers placed near together, with their bulbs as at A and B, of which A is naked but B is covered with soft cotton cloth, from which there passes a band of a few soft cotton threads to a small vessel of water below the instrument. The bulb B has thus always a wet covering around it, from

which evaporation is continually going on, unless
the air is saturated with moisture. As the vapour
rises from the covering of B it absorbs the heat
necessary to convert it from a liquid into vapour,
and the temperature of B falls according to the
evaporation going on; and thus is generally
below the thermometer A. The difference of
the temperatures of the two thermometers, or of
the points a and b in the figure, thus shows
the effect of the wetness or dryness of the air.

Fig. 72.
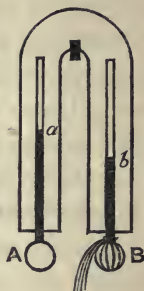
" The difference between the readings of the dry bulb and
wet bulb in this country, between the months of April and
September, will frequently be 9° to 12°, less frequently 12° to 15°,
and occasionally will amount to 18°; and during the other months
of the year it will frequently be between 4° and 9°."

CHAPTER VI.

ON THE RELATION OF LIQUIDS TO GASES AND SOLIDS.

WHEN a vessel of water is placed under the receiver of an air-pump and the pump is worked, we see, as the exhaustion proceeds, that numerous air bubbles form in the water, and burst on rising to the surface. It is found that about $\frac{1}{50}$.th of its bulk of air escapes from ordinary spring water under the air-pump vacuum. This contained air also escapes with the steam in the boiling of water; and hence the need of the air-pump of the condensing steam-engine to remove the air as well as the water from the condenser.

Springs are found in various places in which the water is strongly impregnated with sulphureted hydrogen gas, others with nitrogen gas, others again with a large proportion of carbonic acid gas. From these waters generally the gas escapes when the pressure to which it has been subject is removed. The necessity of boiling mercury before using it in the construction of a barometer has been before mentioned.

The following laws have been established by Henry and Dalton.

1. The gas in a liquid is retained by the external pressure, and when this pressure is removed the gas escapes. It is also in a great measure expelled by boiling.

2. The pressure arising from one species of gas or vapour will not retain another gas in the liquid, for a portion of the absorbed gas escapes until there is equilibrium: the proportions in and out of the liquid having the particular ratio for each gas.

3. The quantity of gas absorbed by a liquid is proportional to the pressure; and the temperature of the liquid rises during the absorption.

4. The absolute quantity of gas which a liquid will absorb under any given pressure is very different for different gases.

Thus water absorbs its own bulk of carbonic acid, and nitrous oxide, $2\frac{1}{2}$ times its bulk of sulphureted hydrogen gas, about $\frac{1}{27}$th of its bulk of oxygen gas, about $\frac{1}{40}$th of its bulk of nitrogen, and $\frac{1}{50}$th of its bulk of hydrogen gas.

5.   Different liquids absorb the various gases in very different proportions.   Thus alcohol is found to absorb twice its bulk of carbonic acid, whilst water absorbs only its own bulk.

In verifying these laws, pure liquids free from air or gas at the commencement are needed, and brisk agitation with the liquid, with fresh supplies of the gas added as the absorption goes on, in a proper apparatus, is required in order to find the effect.

Capillary Attraction and Repulsion.

When a solid body is partly immersed in a liquid, they are found at the places where they come together to have an action upon each other, which is said to be due to capillary attraction or repulsion, arising from forces which are sensible only at insensible distances.

If a flat plate of glass whose perpendicular section is *CD*, fig. 73, be placed vertically in water of which the surface is *AB*, it is found that the water will rise around it to some height as *ab*, about $\frac{1}{7}$th of an inch above *AB*, or the water is said to have a capillary elevation as it approaches the glass.

Fig. 73.
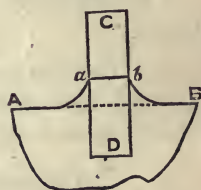
If a flat plate of glass whose perpendicular section is *CD*, fig. 74, be similarly immersed in mercury of which the level of the surface is *AB*, then the surface of the mercury becomes convex near the glass, and is only in contact with it at some depth *ab*, about $\frac{1}{17}$th of an inch below *AB*, so that the mercury near the glass is said to have a capillary depression.

Fig. 74.
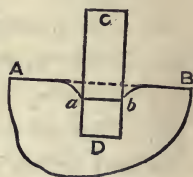
There is said to be a capillary attraction between water and glass, and a capillary repulsion between mercury and glass, but in both sensible only at insensible distances, since the capillary actions cease at the slightest separations of the liquids from the solids. Wax and greasy bodies show capillary depression when partly immersed in water, whilst the metals which are wetted by mercury show capillary elevation when immersed partly in it; so that the terms capillary attraction and repulsion are only relative, there being only attraction between the atoms of dense matter directly.

When a solid body is wetted by a liquid, we conclude that the attraction of the solid for the particles of the liquid is greater than their attraction for each other, so that some portion of the liquid adheres to the solid. When a solid body is not wetted by a liquid, the attraction of the particles of the liquid for each other is greater than their attraction for the solid. The former case of the wetted surface will always give capillary elevation; but the latter case does not necessarily give capillary depression, as will be seen below.

PROP. 64. *To show that if the attraction of a solid partly immersed in a liquid for a particle of the liquid at the surface in contact with it, is more than half that of the liquid for the same particle, there will be capillary elevation; and if less, there will be capillary depression.*

Let *CAD* be the surface of the solid in fig. 75, which is partly immersed in a liquid of which the surface *AB* is in the first instance supposed horizontal, and *A* the particle in the surface in contact with the solid. Since the forces we have to consider are sensible only at insensible distances, if we take an indefinitely small wedge in the solid whose section is *aAb*, and its edge at *A* perpendicular to the plane of the figure, its attraction on the particle

Fig. 75.
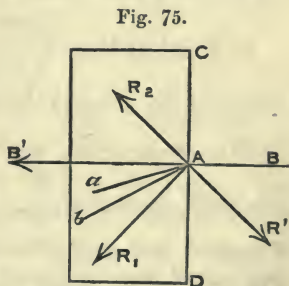
A will be the same in whatever direction it be taken from A in the solid, and the sum of the equal attractions of all such wedges makes up the attraction of the solid. Taking BAB' a straight horizontal line, and CAD vertical, the resultant attraction of the portion of the solid whose section is $B'AD$ will make an angle of 45° with the horizontal and vertical directions, let it equal R acting in AR_1. In the same way the resultant attraction of the portion whose section is CAB' will be R, acting in a line AR_2 whose direction makes an angle of 45° with the lines AB' and AC, as in the figure.

In like manner, again, the resultant attraction of the fluid, whose section BAD will be in the direction AR', making the angles $R'AB$ and $R'AD$ each 45°. Let its magnitude be R'. The particle A being supposed to be in equilibrium, by resolving horizontally we have

$$(2R - R') \cos 45° = 0$$

therefore $2R = R'$.

The resultant of the forces acting in AR_1, AR_2, and AR' is therefore vertical, as well as the fluid pressure and the force of gravity; and hence the surface of the fluid is horizontal, since, as in Prop. 4, the resultant force is always perpendicular to the surface of a fluid when there is equilibrium.

If R is greater than $\dfrac{R'}{2}$, then the resultant lying nearer always to the greater force, and being perpendicular to the capillary surface, this surface will be concave, as in fig. 73.

If R is less than $\dfrac{R'}{2}$, then the resultant will be nearer to the line AR', fig. 75, and the surface will be convex, as in fig. 74.

The angle BAD, fig. 75, is called the angle of contact, and equals 90° when $R = \dfrac{R'}{2}$. Between mercury and glass the angle of contact is found to be about 140°. Between water and glass it is very small, since glass is wetted by water, and R greater than R'.

When two plates are brought near together in a liquid, the form of the surface of the liquid between them is nearly circular; and when a tube of small bore is placed in the liquid, the surface of the liquid within is nearly spherical. The capillary elevations or depressions for different distances of the plates or different radii of the tubes can be found from the consideration that the force exerted is proportional to the line of contact of the liquid and solid, and it is measured by the weight of the column of liquid which is supported above the level of its general surface.

PROP. 65. *To investigate the law of the ascent of a liquid in small tubes of different radii.*

Let *CD* represent a tube, as of glass, in fig. 76; let *AcB* be the level of the surface of the liquid which wets it. Let *ab* be the capillary surface inside the tube, of which the diameter $ab = 2r$, and $ac = h$ the height of *ab* above the level of *AB*. Let *P* be the power due to a unit of length of the surface of contact, then the power exerted within the tube

Fig. 76.

$$= P \cdot 2\pi r$$

and this is balanced by the weight of the column supported above the level of *AcB*. Let ρ be the density of the liquid, and *g* the force of gravity, then the weight of the column

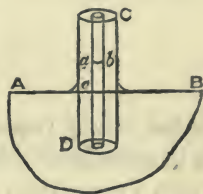$$= g\rho \cdot \pi r^2 \cdot h, \text{ very nearly;}$$

and equating these expressions, we have

$$h = \frac{2P}{g\rho r} \propto \frac{1}{r}$$

or the height varies inversely as the radius of the tube, which is in accordance with experiment; and it is found for water and glass that the height h is one inch when r is $\frac{1}{50}$th inch, from which the heights for other radii can be calculated. It is from

the result so conspicuous in small or capillary tubes that the name of capillary attraction has been given.

PROP. 66.   *To investigate the law of the ascent of a liquid between two parallel plates near together.*

Let *CD* and *EF* represent the two plates in fig. 77, whose distance is *d*. Let *ab* be the surface be-
tween them, and *ac* = *h* the height above
the level of the liquid *AcB*. Let *P*, *ρ*, *g*
be as before.   Let *l* = the breadths of the
plates and length of the line of contact on
each.   Then equating the power along the
line of contact to the weight of the ele-
vated column, we have

Fig. 77.
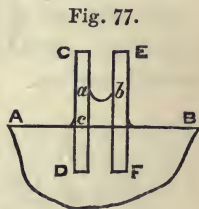
$$2Pl = g\rho h.d.l, \text{ very nearly};$$

$$\text{or } h = \frac{2P}{g\rho d} \propto \frac{1}{d}$$

or the height varies inversely as the distance of the plates. This is in accordance with experiment, and with the result of the previous Proposition; the height being the same when the radius of the tube equals the distance of the plates.

COR.   When there is capillary depression the converse rules hold good.   The depression of mercury is an inch between parallel plates of glass when their distance is $\frac{1}{133}$rd of an inch, and in a tube when its diameter is $\frac{1}{67}$th of an inch, from which the results for other cases can be calculated; and it must be taken into account when barometers are made with tubes of glass whose internal diameters are not very large.

PROP. 67.   *Two plates meet in a vertical line, and are in-clined at a very small angle; required the form of the capillary surface when their lower edges are immersed in a liquid which wets them.*

Let *AOB*, fig. 78, be the vertical line in which the plates *CABD*, *EABF* meet at the small angle *CAE*.

Let *OMH* be the level of the surface of the liquid, and axis of *x*, with *OA* the axis of *y*. Let *aPb* be the surface of

Fig. 78.
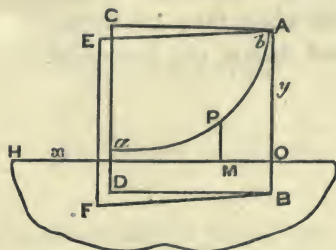
the liquid between the plates, and *P* any point, drawing *PM* parallel to *OA*; let *OM*=*x*, *PM*=*y*. Now if *D* is the distance of the plates at a unit of distance from *O*, and *d* is the distance at *P*,

$$\text{we have } \frac{D}{1} = \frac{d}{x} \text{ or } d = D \cdot x$$

and the height *PM* = *y*, by the last Prop., $\propto \frac{1}{d} \propto \frac{1}{Dx}$,

and $xy = \text{constant} = m$, say,

which is the equation to the rectangular hyperbola referred to the asymptotes, as seen in experiments.

From the properties above discussed it arises that when we dip a needle in a liquid that wets it, the drop which hangs from the needle when withdrawn from the liquid takes a position of equilibrium, as at *A*, fig. 79, and does not fall to the point.

Fig. 79.

When a liquid which wets a substance is placed between two plates of it, as at *A* in fig. 80, it has concave surfaces and moves up to the angle where they meet; and if small in quantity they may be turned with the open part downwards without its falling out.

Fig. 80.

When there is capillary repulsion be-
tween the substance and liquid, a small
quantity of the latter between plates, as
at *A*, fig. 81, takes a flattened convex
form, and falls out when they are turned
with the open end below the horizontal
direction.

Fig. 81.

CHAPTER VII.

ON THE MOTION OF FLUIDS.

THE problems of hydrodynamics which can be solved without the aid of the differential calculus are not very numerous.

In fluids the constituent atoms being free to move independently of each other on the application of the slightest force, except that in liquids the attraction of aggregation requires to be considered for the strict solution of most cases, the motion of the fluid must be considered as originating in the motion of its atoms individually. In the motion of a fluid, when no crystallic arrangement like that in water near the freezing temperature exists, the laws of fluidity in regard to the symmetrical arrangement of the atoms have to be recognized. If circumstances different to those which would exist in the case of equilibrium are impressed upon one or more atoms of a fluid, then motion must ensue; and it may be a motion of translation of the atoms amongst each other or with respect to other bodies, involving a motion of masses of the fluid; or it may be that a vibratory motion of the nuclei or centers of the atoms about their places of rest only exists, whilst the most general and most frequent case will involve both motions of vibration and translation. Admitting that both kinds of motion do really exist, it requires, in most problems, that we take notice of one only as affecting the result which is under investigation.

A force may act upon a whole mass of fluid, and yet a part of it only may receive motion at any instant, because the action of the force upon the other parts is counteracted by the resistance of the vessel in which the fluid is contained. The case of a condensed gas in a receiver with an aperture in it which is

opened, and the case of water flowing from an opening in the
containing vessel, are such cases: the portion at and near the
opening at any time being that which receives motion at any
instant.

Torricelli first ascertained the law of the velocity with which
a liquid flows from an opening in a
vessel which contains it. Let AB
be the surface of the liquid in the
vessel, fig. 82, C a small opening
in an arm at the lower part; then
when the jet from C was vertical,
as in the figure, it attained a height
at the highest point a nearly the
level of AB, and he concluded that
if the resistance of the air and fric-
tion had not existed it would have
attained accurately to the level of
AB, and therefore when it issued
from the aperture at C it must have had a velocity equal to that
which a heavy body would acquire in falling from the level of
AB to the point C; for it is known in the science of dynamics,
that when a heavy body is projected directly upwards it loses
the velocity in ascending which it acquires in descending again.

Fig. 82.

Let h be the height of AB above C, v the velocity of issuing
at C, and g the force of gravity; then by dynamics we have

$$v^2 = 2gh.$$

Since fluids transmit pressure equally in all directions, the velo-
city of the jet at any orifice will be the same whether it issues
vertically upwards, vertically downwards, horizontal or inclined,
being always that due to the height of the surface above the
orifice. This law has been found to hold more accurately for the
gases than for liquids, as might be expected from their more
perfect fluidity. The gas contained in a vessel or receiver having
a small aperture which is suddenly opened into a vacuum, the
velocity of the issuing jet is that due to the height of the column
of the gas, which, if of uniform density, would produce the
pressure or elastic force which it exerts in the receiver. Thus
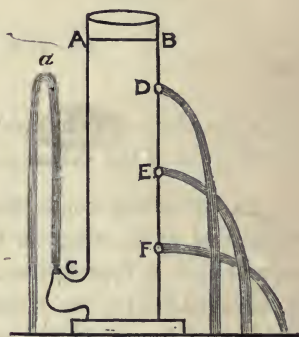
hydrogen gas issues into a vacuum with much greater velocity than atmospheric air, and carbonic acid gas with less. Professor Graham found that if the gas issued into a space containing a moderate portion of gas instead of into a vacuum, a similar law held good, but with a height of the gas due to the difference of the pressures inside and outside the receiver.

It was a long time after Torricelli's discovery before the science of hydrodynamics was sufficiently advanced to afford a strict mathematical proof of the above law; and as it involves the differential calculus, it cannot be admitted here.

When a liquid issues *into the atmosphere* from an aperture in a *thin plate*, the jet is found to contract after leaving the aperture, and the narrowest part is called the *vena contracta*. It is found that if the area of the *vena contracta* be taken as the effective orifice, the quantity of liquid which issues in a given time comes near to that given by calculation. Newton considered the area of the *vena contracta* to bear the ratio of 1 to $\sqrt{2}$ to that of the real aperture, but it is generally considered to be that of 5 to 8. When to the aperture a cylindrical, conical, or other form of tube is fixed, it is called an *adjutage*, and its form affects considerably the quantity of liquid discharged in a given time.

When the velocity of a fluid is the same at the same point at all times, it is called a case of *steady motion*. When the velocity is not always the same, it is called a case of *variable motion*. In Prop. 67 the motion is *steady*, in Prop. 68 it is *variable*.

PROP. 68. *A cylindrical vessel containing liquid, having a small aperture in the base, to find the time of a quantity flowing out which is equal to the content of the vessel, when it is kept continually full.*

Let CB be the surface of the fluid in the vessel in fig. 83, A the aperture in the base, let the height of BC above A be equal to h; then if v is the velocity of the fluid issuing at A, we have

$$v^2 = 2gh.$$

Let α be the area of the *vena contracta* of the
issuing stream, s the space any part would move
through in a time t if v were constant; then the
quantity flowing out in the time t is

$$s \times \alpha = vt.\alpha$$

$$= t\alpha\sqrt{2gh}.$$

Fig. 83.
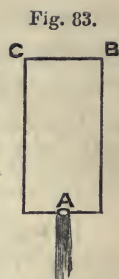
Now if r is the radius of the cylindrical vessel with the height
h, we have its volume $= \pi r^2.h$, and, equating this to the last
expression, we have

$$t = \frac{\pi r^2}{\alpha} \sqrt{\frac{h}{2g}}$$

which gives the time t as required.

PROP. 69. *To find the time the vessel of the last question will
take to empty itself when no fresh liquid is added.*

As the surface CB descends, the velocity of issuing at A will
be continually diminishing until the vessel is empty, and will
be always that due to the height of the surface above A. Now
when a heavy body is thrown vertically upwards, its velocity at
any point in the ascent equals that at the same point in the
descent, namely, that due to the height fallen through in the
descent; and the time occupied in ascending to the highest point
equals that of descending again to the same point. In dyna-
mics we have the space s fallen through by a heavy body $= \frac{1}{2}gt^2$
$= \frac{1}{2}vt$, if v is the velocity acquired in falling for a time t from
rest;

$$\therefore \ t = \frac{2s}{v}.$$

But with a constant velocity the space described $=$ vel. $\times$ time,
and

$$\text{the time} = \frac{\text{space}}{\text{velocity}}.$$

Therefore the time of the ascent or descent of the body acted
on by the force of gravity equals twice the time required to pass

through the same space if the body moved with the first or last velocity continued constant.

Hence, comparing with the cases of the emptying vessels of liquid, the time of the cylindrical vessel emptying when no fresh liquid is added equals twice that of the same quantity flowing out when it is kept always full, for the velocity of the surface in the vessel $= \dfrac{a}{\pi r^2} \times$ velocity at the orifice.

PROP. 70.   *When a jet of liquid issues from an aperture in a vessel containing it, to find the form of the jet.*

If the jet be vertical it will remain vertical, but if it issues in any other direction the curve which it takes will be the common parabola, since each particle of the liquid may be considered as projected from the orifice, and its path will be that of a projectile, as investigated in dynamics. For the directions of projection horizontal the forms of the jets at D, E, and F are represented in fig. 82. The maximum range on a horizontal plane through the orifice occurs when the jet issues at an angle of 45° with the horizon, as found by theory and experiment.

When any mass of fluid is in motion it possesses momentum as a solid would, but its effect upon any obstacle it should strike would be very different, from its wanting the attraction of cohesion possessed by the solid. If the fluid is moving in a pipe, the momentum may be easily seen to produce a very great effect when suddenly checked, since it is restrained from diverging laterally by the resistance of the pipe. When a stop-cock upon a pipe from a cistern is left open some time, until the water in the pipe has acquired its full velocity, and is then suddenly shut, we hear a succession of blows within the pipe, which may burst it, if it is weak. This property was made by Montgolfier, in his hydraulic ram, the means to raise water from a lower to a higher level by very simple machinery. This machine is still in use in many places, performing the work for which it was invented.

PROP. 71.   *To explain the construction and mode of action of Montgolfier's hydraulic ram.*

Let *AB*, fig. 84, be the level of the surface of the water in the cistern, from which the pipe *CD* passes and terminates in a chamber which has two openings fitted with strong metal valves, as at *a* and *b*. Of these *b* opens downwards, and *a* opens upwards into an air-vessel, into which the exit-pipe passes air-tight, to near the bottom. If the valve *b* is shut the pressure of the water in the pipe will raise the valve *a*,

Fig. 84.
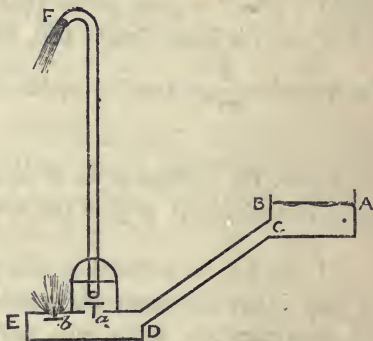
condense the air in the air-vessel, and find its level with *AB* in the exit-pipe.   If the valve *b* be now pushed down a jet of water will issue through the opening and the water in the pipe will acquire a certain velocity and momentum, and when this becomes sufficient, it will lift the valve *b* and close the opening suddenly.   The momentum of the water will now raise the valve *a*, and a portion of water will enter the air-vessel, condensing the air still more, and then the valve *a* will fall and prevent the water escaping again, when the elastic force of the condensed air will force it up the exit-pipe until it escapes at *F*. When the water in the pipe has come to rest the valve *b* will fall again and the same result take place as before.

PROP. 72.   *To explain the construction and mode of action of Barker's mill.*

Barker's mill has a vertical pipe *AB* connected with a horizontal one *CD*, fig. 85.   They turn together about a pivot *E* and in a bearing at *F*.   The horizontal pipe *CD* is closed at the ends but has two openings at *a* and *b*, at the opposite sides of it.   When water is poured into the upper part and escapes through the openings *a* and *b*, with a velocity depending upon the height of the surface in the pipe *AB*, there is an unbalanced

reaction opposite each of the holes
a and b, and these cause rotation
in the opposite direction to that of
the issuing streams.

Fig. 85.

Moving fireworks generally act
upon the same principle with Bar-
ker's mill; that is, an unbalanced
reaction from the heated gases which
issue from the openings in them,
gives them motion in the opposite direction. The rocket as-
cending with great velocity requires a very strong case to con-
tain the composition with which it is charged, especially as the
case is choked near its lower end and a large surface of the com-
position is on fire at the same time, so that the produced gases
issue with great velocity and produce a large unbalanced reaction.

The Turbine is a modification of Barker's mill, with a hollow
horizontal chamber between two circular discs in place of the
horizontal pipe. This chamber has openings along its circum-
ference and partitions of a particular curved form in its interior.
It is now constructed so as to exhaust the power of a stream of
water as completely as the best water-wheels, and can be used
with a supply of water to which they could not be applied.

The *undershot* water-wheel consists of a vertical wheel turn-
ing about an axle, with boards attached to its circumference
perpendicular to its plane, and called float-boards. These float-
boards dipping at their lowest positions into a stream of water
are carried along by its momentum, which thus gives rotation
to the wheel.

The *breast-wheel* has a trough of masonry of the form of the
wheel, in which the water strikes the float-boards above the
lowest point, and acts partly by its weight and partly by its
momentum.

The *overshot* wheel has the water laid on the upper part of
the wheel; which thus acts chiefly by its weight. It is some-
times considered identical with the *bucket-wheel*.

The *bucket-wheel* has a series of troughs formed along its
circumference of peculiar shape to retain the water as long as
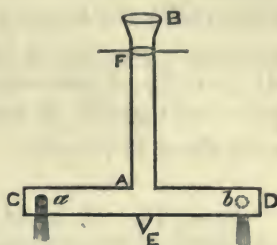
possible during the revolution of the wheel. The water is laid on, or enters the troughs, near the upper part of the wheel, and acts almost entirely by its weight. This is the most effective of all the forms of water-wheels, and exhausts most completely the power of the fall of water, of which it renders available for work about 75 per cent.

PROP. 73. *A vessel turning about a vertical axis contains a quantity of liquid; required the form which the surface of the liquid assumes.*

Let $ABCD$ in fig. 86 represent the vessel rotating round the vertical axis EF, and containing liquid. It is found that the surface of the liquid, which was horizontal before the rotation commenced, becomes curved and highest at the side of the vessel after it has commenced. The rotation having become uniform, the surface of the liquid takes a form such as bac, which remains the same, and therefore there is equilibrium amongst the forces acting upon the liquid. Now the forces acting upon any particle are the fluid pressure, the force of gravity, and the centrifugal force arising from the rotation.

Fig. 86.
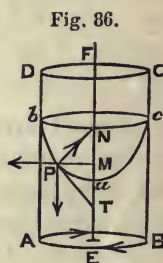
If P be a particle in the surface bac, and we draw the normal PN, the tangent to the curve PT, and the radius of the circle which P describes PM, then the pressure of the atmosphere and the fluid pressure both act in the normal PN; in which direction the resultant of the vertical force of gravity g, and the horizontal centrifugal force $= \dfrac{(\text{velocity})^2}{\text{radius}} = \text{radius} \times (\text{angular velocity})^2 = a^2 \cdot PM$ must also act, if $a =$ angular velocity of rotation.

Resolving in the direction of the tangent PT, we have for equilibrium

$$a^2 \cdot PM \sin PTM - g \cos PTM = 0;$$

or $\qquad\qquad \tan PTM = \dfrac{g}{a^2 \cdot PM} \propto \dfrac{1}{PM},$

which is the property of the parabola. Let $y^2 = 4mx$ be its equation with origin a and axis of x the axis of rotation; then by conic sections $\tan PTM = \dfrac{2m}{y}$, therefore, comparing with the above equation, we have the semiparameter of the generating parabola of the surface equal to $\dfrac{g}{a^2}$.

PROP. 74. *To explain the cause of the rarefaction in diverging streams of fluids.*

The discovery of the remarkable effects of diverging streams of air and their explanation were made by Mr Roberts of Manchester. An instrument for showing these effects is often made, as in figures 87 and 88, of tin-plate; A being a circular disc, say 1 inch in diameter, and BC another equal disc perforated in its center to admit the end of the pipe D, which is soldered to it. Some small prominences are generally made round the edge of BC to prevent the disc A sliding laterally away.

Fig. 87.

Fig. 88.

When a stream of air is forced along the pipe D, the disc A is not then blown away, but takes at some small distance a position of equilibrium, from which it requires some force to remove it.

Fig. 89.

This effect arises as follows: let a be the opening of the pipe in the disc CB, fig. 89, and let the stream of air diverge on all sides from it as it issues from the pipe D on striking the disc A; then in the divergence it becomes rarefied, and at a short distance from a becomes of less density than the atmosphere. Let bac represent the sector which a portion of air would describe on passing from the center to the circumference of the disc BC; we see that if by the first law of motion, the air continued to move with the velocity with which it issued from a, an elementary portion, such as pq, would soon occupy many times the space
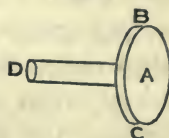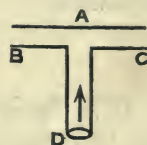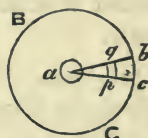
which it did on leaving the opening a, and become more rarefied as it was further from the center. If there were a vacuum around the edge, this would be really the case; but with the atmospheric pressure at the edge the velocity is checked gradually, and a certain amount of rarefaction only takes place. When the disc A is in a position of equilibrium, the atmospheric pressure on its outer face equals the sum of the variable pressures on the inner face.

Cor. If a stream of air is forced along a conical pipe, a like result arises if it moves from the small to the wide end, or it becomes rarefied as it passes along. If, however, it passes from the wide end towards the small end, the converse of condensation takes place.

The effects of adjutages to apertures in vessels of liquids which empty through them, depend on the same principles, and the important subjects of the best forms of the chimneys of furnaces, the construction of safety valves, and the forms of pipes for conveying steam or liquids, involve the same considerations.

On Waves.

If a disturbance is impressed on the surface of a liquid at rest, there are waves formed around the place where the disturbance is made. The simplest case is when the waves are formed around a center, as when a stone is thrown into a pond of still water.

Suppose a heavy round body, as a, fig. 90, to be thrown into still water, it will carry before it the water at the surface where it falls, and on account of the little compressibility and high degree of elasticity of water, elevations as b, c will be formed around a. Then the heavy body falling below the surface, the water which had been depressed below the level rises above it at d, whilst depressions take the places of the elevations at b and c, and fresh elevations arise at e and f, and so onwards;

Fig. 90.
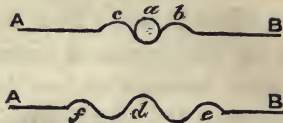
so that a series of circular waves diverge around the center, whilst the space within continues for some time in a state of oscillation and undulation. If we observe the surface at any point by watching some small floating body, we see that each point in the surface has an oscillatory motion, rising above and falling below the original level of the surface alternately, and thus gives rise to an undulatory or wave motion diverging from the center.

If two sets of waves be formed by throwing two bodies into water, it is easily seen that they exist together, but interfere whilst passing through each other, so that where each set would cause an elevation we have produced a higher elevation, and where each would produce a depression we have a deeper depression produced; but where one set would produce elevation and the other depression, we have a less elevation or depression, and if the tendencies were equal the surface remains level. When two stones are thrown *simultaneously* into a pond of still water, the waves being alike round each center, at each instant the interference is regular, so that the places where they strengthen each other, and the places where they neutralize each other, are seen to occur in regular hyperbolic curves.

If a series of waves strike directly against a plane surface they are reflected, and the series of reflected waves pass through the original waves as if they came from an origin at an equal distance behind the plane to what the real origin is in front of it.

On Sound.

Sounds are affections of the organ of hearing, which may arise in various ways, but the most frequent are those which arrive through the air.

That the air is the medium through which the sound from a distant sounding body comes to the ear, is shown by an experiment with the air-pump; for if a bell be rung in the receiver, the sound diminishes as the exhaustion is produced, and at length in the best air-pump vacuum is scarcely audible; so

that we conclude no sound could be transmitted through a perfect vacuum.

A sound may be produced by a single impulse transmitted through the air, as when a body is struck by a hammer; but a continuous sound requires a succession of impulses for its production, and when these occur with regularity they constitute a musical sound.

That sounding bodies, such as bells, springs, stretched cords, &c. are in a state of vibration whilst sounding can be shown by various experiments. A simple and beautiful experiment with Wheatstone's kaleidophone shows this for springs producing a musical note, in an instructive manner. The kaleidophone consists of a disc of wood AB, fig. 91, into which are stuck wires of about $\frac{1}{20}$th inch in diameter and 9 to 12 inches long or more, of which one should be bent at right angles, as in the figure. A small convex mirror is stuck at the free end of each by black sealing-wax, as at a, b and c. What are called steel beads, but are really silvered glass, may be used as convex mirrors, which answer the purpose. A string of these beads can be often purchased at the cheap toy shops for a few pence.

Fig. 91.
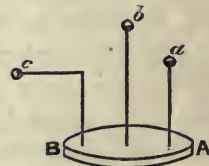
When this instrument is placed in the sun's light we see a minute image of the sun in the convex mirror; and if the wire be struck sharply we see when it is straight that the image forms vibrations of changing ellipses, and when bent at right angles changing lemniscates. These may exist without sound being heard, but when a violin bow is applied near the fixed ends of the wires to produce musical notes, there arise small vibrations upon the larger ones. These are called superimposed vibrations, and are produced by the vibrations of the wire giving the musical note. They appear variously upon the larger ones, and often look like the teeth upon a fine saw, being finer and closer as the note produced is higher.

We learn from such experiments that the form of the vibrations may be very various, but the frequency of the vibrations changes as the musical note changes in pitch.

The vibrations of the wires produce vibrations in the atoms of air near them, and these are transmitted by the elasticity of the air to other atoms at a distance, and thus form sound waves diverging around the sonorous origin.

These sound waves travel through the air with a velocity of about 1100 feet per second, which can be shown by noting the interval of time between the flash and report of a gun at a known distance. By a comparison of various experiments, it is concluded that the velocity of sound over the ordinary surface of a country is 1090 feet per second at the freezing temperature of water; and that it is 1·14 foot per second more for every degree of temperature Fahrenheit, above freezing, and the same quantity less for every degree below it. The velocity of sound does not change with changes of the barometer, because the density of the air changes with its elastic force. The velocity of sound is the same for notes of different pitch; for we hear the notes from a peal of bells at a distance in their regular order, and in like manner the tune played by a band of musicians sounds the same at different distances.

Ordinary sounds cease to be heard over very moderate distances of the rough surface of a country, but are audible very much further over still water or smooth surfaces. It is probable that sound is not only lost in passing over rough surfaces, but that its velocity is slightly retarded, as waves at the surface of water are in passing up a rough channel. In artillery practice, where the distance of an object to be fired at is often found by the interval between the flash and report of a gun, it is found that a little more elevation must be given to the gun when fired over water than when fired over land, since "*the water attracts the shot*" more than the land! A more philosophical explanation is, that the velocity of the sound was rather more over the water than over the land, and hence the distance it would travel in the same time rather more.

The history of the mathematical investigation of the velocity of sound is singular. Sir Isaac Newton, from insufficient methods, found that the velocity of sound in air was equal to $\sqrt{g \cdot H}$, where g is put for the force of gravity, and H the height the earth's atmosphere would reach if homogeneous. This is the same

expression as is found by the more correct solutions of Euler and Lagrange; and it gives the velocity of sound to be about 916· feet per second, or nearly $\frac{1}{6}$th less than that found by experiment. Laplace suggested that the discrepancy arose from heat being suddenly developed in the rapid condensation of the air; and Poisson, assuming a particular form for such effect, found that the velocity is represented by the expression $\sqrt{g \cdot \gamma \cdot H}$, if $\gamma = \dfrac{c}{c_1}$, where c is the specific heat of air under a constant pressure, and c_1 the specific heat under a constant volume. By methods liable to serious objections the values $\dfrac{4}{3}$, 1·3748, 1·4061, 1·421 have been found for γ. The author, from other methods, found that $\gamma = 1$. for small condensations and rarefactions; and he has shown that if we consider the condensations and rarefactions to take place in all directions when the sound wave passes through air, that the expression for the velocity of sound is $\sqrt{\dfrac{3}{2} g \cdot H}$. This gives the velocity $= 1122 \cdot 2$ feet per second at the freezing temperature; and from the experience of the artillery, it is probably the true velocity when sound passes over still water.

The expression, velocity $= \sqrt{g \cdot H}$ supposes the condensations and rarefactions to take place only in the direction of the wave motion. This supposition gives very nearly the velocity of sound passing through water, as determined by MM. Colladon and Sturm, in their experiments through the water of the Lake of Geneva, who found it to be 4708 feet per second. Using Canton's and Œrsted's value of the compressibility of water, the velocity is found to be by calculation 4876 feet per second, which is not greatly different from experiment.

If we note what takes place when a flat spring or the side of a bell is in a state of vibration producing sound, we see that a series of atoms of air which were originally at equal distances, as in the line ab, fig. 92,

Fig. 92.

will be put into states of alternate condensation and rarefaction, as at *cd* and *ef* respectively, and the vibrations will be in the direction in which the sound travels. The vibrations are from this called longitudinal vibrations. The distance from *c* to *d* or from *e* to *f* is called the breadth of the sound wave, and the letter λ is frequently put for it. It is convenient to compare these waves with those formed at the surface of water, though so differently produced; for instance, in fig. 93, *ad* is the breadth of a wave $= \lambda$, and the particles at *a* and *d* are in the same state or phase of vibra-

Fig. 93.

tion; therefore if *t* be the time of an atom describing a complete vibration and of the wave travelling from *a* to *d*, *v* the velocity of sound $= 1100$ feet per second, nearly, we have $\lambda = v \cdot t$, from which either λ or *t* can be found when the other is given.

Vibrating columns of air in wind instruments, and vibrating bodies generally, have what are called *loops* and *nodes*. The meanings of these terms are perhaps best illustrated by their places in vibrating strings fixed at the ends, as *AB*, fig. 94. It may vibrate as a whole or between *a* and *b*, and then produces its lowest or fundamental note, or it may vibrate as between *c* and *d*, in two halves with a point *e* stationary, and then called a *node*, whilst the points *p* and *q* are in the state of greatest vibration, and are called *loops*. The

Fig. 94.

note then sounded is the octave of the fundamental note. The same string may also vibrate as between *f* and *g*, with loops at *r*, *s*, and *t*, and nodes at *h* and *l*. It then sounds the fifth note above the octave, and so onwards.
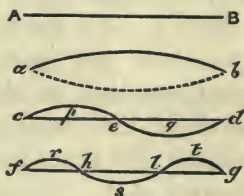
The places of the nodal lines upon plates of glass are seen in a great variety of forms by putting dry sand upon the plate and causing it to sound, when held at some point, by applying a violin bow to the edge.

The pitch of a note depends on the frequency of the vibrations,

or it is inversely as the time of a vibration, or as $\frac{1}{t}$, and therefore as $\dfrac{1}{\dfrac{\lambda}{v}}$; but v is constant, therefore it varies as $\frac{1}{\lambda}$.

PROP. 75. *To find the pitches of the notes which can be obtained from the same stretched string of given length.*

When the tension of a string is the same, the time of vibration of the lowest note is proportional to its length; and we saw that the string AB, fig. 94, might vibrate as a whole, in halves, thirds, &c.; and therefore from the same string the notes whose pitches are as 1, 2, 3, 4, &c. can be produced; and by the property of superimposed vibrations two or more of these may be sounded together.

PROP. 76. *To find the pitches of the notes which can be obtained from a cylindrical tube closed at one end and open at the other.*

Let AB represent, in fig. 95, the tube, closed at A and open at B. When we blow across the open end the air will be set in vibration, and there will be a loop in the column of air in the tube at B; but the air at the closed end must be at rest, and there must be a node at A. Therefore the sound wave of the lowest note will have for its breadth four times the length of the tube or distance AB.

Fig. 95.

Any notes may be obtained from the same tube which fulfil the required conditions of a loop at B and a node at A; as, for instance, in fig. 96, there may be nodes at A and a with loops at B and b. The sound wave will have its length one third that of fig. 95. In the same way the conditions will be fulfilled when the sound wave has its length one fifth, one seventh, &c. of that of the fundamental one, and the pitches of the notes which can be obtained will be as 1, 3, 5, 7, &c.
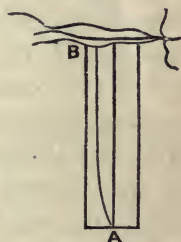

Fig. 96.
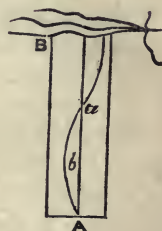
We see that by blowing stronger over the tube the next note to be obtained above the fundamental one is the fifth above the octave, and the octave cannot be obtained; this is in accordance with experiment.

The closed organ pipes or stopped diapason belong to this class.

PROP. 77. *To find the pitches of the notes which can be obtained from a tube open at both ends.*

When the tube is open to the air at both ends, as in fig. 97, and we blow across one end, setting the air in vibration, there will be a loop there and at the other end also, since it is open to the air; but there must be a node between them, as at *a* in the figure. The sound wave of the lowest note is therefore twice the length of the tube, and the note produced is the octave above that which would be produced if the end *A* were closed. This agrees with the experiment.
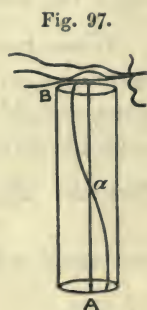
Fig. 97.

The same conditions will be fulfilled if we have loops at *A*, *c*, and *B*, with nodes at *a* and *b*, and the sound wave, as in fig. 98, of the note produced is the length of the tube, being the half of that of fig. 97, and the note produced is the octave of the fundamental note.

Fig. 98.
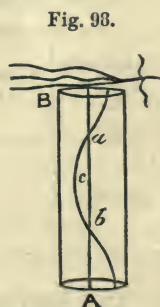
Similarly the conditions will be fulfilled if there are three, four, five, &c. nodes in the tube; and the pitches of the notes to be obtained are as 1, 2, 3, 4, &c., which is in accordance with experiment.

The flageolet, flute, bugle, open diapason pipes of the organ, &c. belong to this class of musical instruments.

The reed stops of organs are formed with vibrating brass springs, and the note produced depends upon the length and elasticity of the spring; but it is modified by the length and form of the tube to which it is attached, so as to imitate the

hautboy, trumpet, violin, &c.   With a seraphine reed and a tube which can be lengthened by sliding a tube over the one holding the reed, we obtain the series of vowel sounds with the same note, and the order is *i, e, a, o, u,* pronounced in the continental manner as *ee, ai, aa, o, u.*   These are formed in the human voice by altering the cavity of the mouth and throat.

The human voice is formed in the larynx behind the prominence in front of the neck, by the vibrations of the tendinous chordæ vocales, which are under the power of muscles acting under the influence of the will.

When two notes are sounded together, they harmonize or please the ear in proportion as the lengths of their waves are commensurable; thus the fundamental note and its octave may be said to form a unison.   The next most perfect accordance is when the ratio is 1 to $\frac{3}{2}$; the next again when it is 1 to $\frac{4}{3}$; the next again when it is 1 to $\frac{5}{4}$; and the next again when it is 1 to $\frac{5}{3}$.

These in the series of natural notes are as follows:

Fundamental note C	Second D	Major third E	Fourth F	Fifth G	Major sixth A	Seventh B	Octave C
pitch 1	.	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$		2

The interval being large between *C* and *E,* the note *D* is made the fifth of the octave below the fifth, and its pitch is $\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$, which though it will not accord with the fundamental note, will do so with the fifth, its best chord.

The interval between *A* and *C* octave is also large, and the note *B* is made the major third to the fifth, with which it will

therefore accord, and its pitch is $\dfrac{3}{2} \times \dfrac{5}{4} = \dfrac{15}{8}$. Thus the diatonic scale of notes is as follows:

C	D	E	F	G	A	B	C
pitch 1	$\dfrac{9}{8}$	$\dfrac{5}{4}$	$\dfrac{4}{3}$	$\dfrac{3}{2}$	$\dfrac{5}{3}$	$\dfrac{15}{8}$	2

The note C is called the key-note of the scale. It is clear that if any other note were taken for key-note, the relations of the sound waves would not hold the same.

In instruments with fixed notes the tuning requires temperament, so that in playing in the most usual different keys the least possible discord may arise; and the ear does not require perfect concords.

A TABLE OF SPECIFIC GRAVITIES.

A cubic foot of water at 60° Fahrenheit weighs 997·137 ounces avoirdupois; 100 cubic inches of atmospheric air, barom. 30 in., therm. 60°, weighs 31·0117 grains.

Specific gravities of gases, air being taken as standard fluid.

Atmospheric air............	1·0000
Hydrogen gas...............	·0688
Nitrogen „	·9757
Oxygen „	1·1026
Carbonic acid gas	1·5245

Specific gravities of some bodies, water being taken as standard fluid.

Water........................	1·0000	Flint	2·59
Sea-water ...... 1·0263 to	1·0295	Cornelian	2·61
Cork	·2400	Rock crystal	2·65
Pine wood	·5500	Emerald	2·77
Maple „	·7550	Calc spar	2·72
Mahogany .... ·820 to	1·063	Diamond	3·52
Ebony	1·209	Topaz, Oriental	4·01
Box wood .... ·912 to	1·328	Beryl „	3·55
Oil of turpentine	·869	Ruby „	4·28
Camphor......................	·989	Garnet, Bohemian.........	4·19
Olive oil	·915	Barytic spar	4·43
Linseed oil	·940	Granite .... 2·65 to	2·75
Alcohol .... ·793 to	·829	Potassium	·865
Nitric æther	·909	Sodium	·972
Sulphuric æther... ·632 to	·739	Aluminium	2·560
Tallow	·942	Antimony	6·702
Bees'-wax	·964	Zinc .... 6·861 to	7·100
Ivory	1·825	Tin	7·291
Gum Arabic	1·452	Iron	7·788
Sulphur	1·991	Nickel	8·279
Phosphorus..................	1·714	Cobalt........................	8·538
Pit coal	1·329	Copper	8·895
		Bismuth	9·822
		Silver .... 10·474 to	10·530
		Lead	11·352
		Mercury	13·568
		Gold............. 19·275 to	19·340
		Platinum	20·980
		Plate glass .... 2·48 to	2·52
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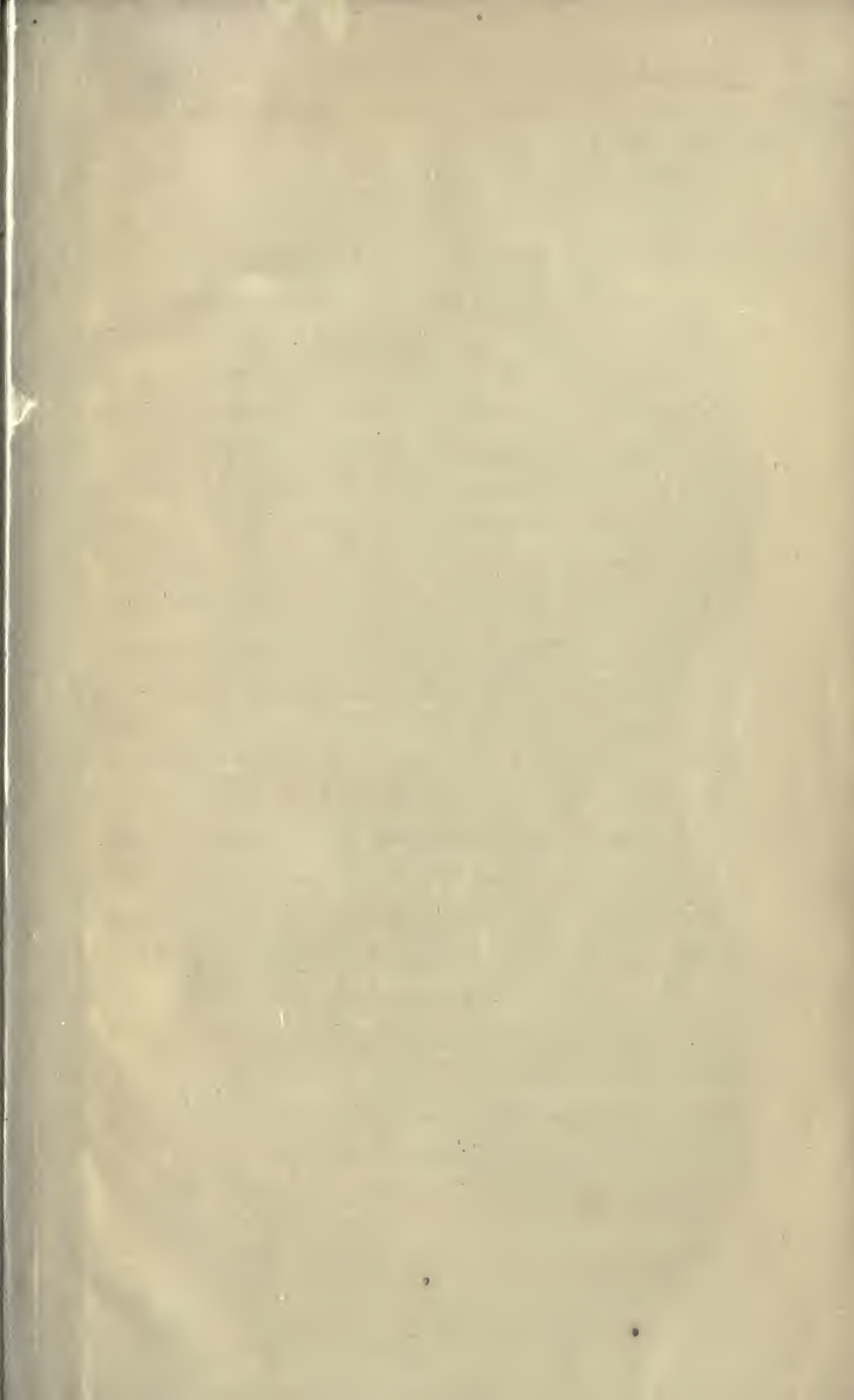
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